The First Czech-Catalan Conference in Mathematics — Abstracts
T. Chudlarský (editor)

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Preface

Dear Participants,

On behalf of the Czech and Catalan Mathematical Societies, we warmly welcome you to our first joint event. An agreement on reciprocal membership and cooperation between our societies was signed less than a year ago, in September 2004, during the Joint Mathematical Weekend with the EMS in Prague. We were both aware of existing strong cooperation between groups of Czech and Catalan mathematicians in our fields of research. Yet, we have been pleasantly surprised by the response of our mathematical communities to the idea of organizing a joint conference. Our modest expectations of four parallel sessions have expanded to six strong ones. And the number of registered participants has grown beyond any expectations. Many world known Czech and Catalan mathematicians are involved in the program and organization of the conference. In the last weekend of May 2005, Prague is going to host an important mathematical event, not only within the perspective of our two societies.

The First Czech-Catalan Mathematical Conference, and our formal cooperation in a broader sense, is following the current trends of collaboration and unification in Europe. But perhaps there is more to it. The enthusiastic response of our members may suggest that our nations are closer to each other than one might expect. Maybe a significant role is played by the similarities in the national spirits and by parallels in the historic development. In any case, we are very much looking forward to further cooperation of our societies.

For the forthcoming weekend, we want to thank all session organizers and members of the organizing committee for the efforts invested in this joint event. We also thank all institutions supporting the event: Faculty of Mathematics and Physics of Charles University in Prague, Institute for Theoretical Computer Science of Charles University, Mathemtical Insitute of the Academy of Sciences of the Czech Republic, University of West Bohemia in Pilsen, Institut d'Estudis Catalans, and several research grants by participating teams. Last but not least, we would like to mention two names explicitly: Oriol Serra (Barcelona), who was the driving force behind the idea of organizing a joint conference from the very beginning, and Jiří Fiala (Prague), for whom the Czech-Catalan Conference became the number one issue in the last weeks, and who is responsible for all good things and achievements of the local organizers.

We wish you all a fruitful and pleasant weekend.

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Plenary talks

Statistical methods for the calibration and validation of simulation models

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From a methodological point of view it is widely accepted that simulation is a useful technique to provide an experimental test bed to compare alternate system designs, replacing the experiments on the physical system by experiments on its formal representation in a computer in terms of a simulation model. Model calibration and validation is inherently an statistical process in which the uncertainty due to data and model errors should be account for. This lecture presents explicit methods to take into account the autocorrelation dependencies between traffic data, and the specific time dependencies characteristics of traffic data whose emulation is one of the main abilities of microscopic simulation. The proposal is illustrated with case studies from applications of microscopic traffic simulation where the calibration of route choice models becomes a critical component, from the analysis guidelines for calibration are also proposed in the route based simulation.

Direct-sum decompositions in additive categories

Alberto Facchini

University of Padova

Recent progress has been done recently in the study of direct-sum decompositions in the category of modules. We will try to extend these results to the setting of arbitrary additive categories, or, better to additive categories in which idempotents have kernels, equivalently, categories in which idempotents split. For these categories, called *amenable* by Freyd, the Krull-Schmidt Theorem holds for objects with a local endomorphism ring, as was observed by Bass. Our main ingredients will be valuations of additive categories and categories with invariant basis number.

Cluster algebras and triangulated categories

Bernhard Keller

Université Paris 7

Cluster algebras were invented by S. Fomin and A. Zelevinsky in spring 2000 as a tool to approach Lusztig's theory of canonical bases in quantum groups and total positivity in algebraic groups. Since then, cluster algebras have become the center of a rapidly developing theory, which has turned out to be closely related to a large spectrum of other subjects, notably Lie theory, Poisson geometry, Teichmüller theory and quiver representations. Recent work by many authors has shown that this last link is best understood using the cluster category, which is a triangulated category associated with every Dynkin diagram. In this talk, I will report on these developments and present the cluster multiplication theorem, obtained in joint work with P. Caldero, which directly links the multiplication of the cluster algebra to the triangles in the cluster category.

An interplay between real functions theory and potential theory

Jaroslav Lukeš

Charles University Prague

For an open bounded set U of $\mathbf{R}^m$, let the space $H(U)$ consist of all continuous functions on $\overline{U}$ which are harmonic on U. Given a continuous function f on the boundary of U denote by f^{CU} the *Dirichlet solution* of f. Further, let $\mathcal{B}_1^b 1(H(U))$ be the space of all bounded functions on $\overline{U}$ which are pointwise limits of functions from $H(U)$.

We show a close relation between some methods of real functions theory and potential theory. For example, we indicate proofs that the Dirichlet solution f^{CU} belongs to the space $\mathcal{B}_1^b(H(U))$. Moreover, we examine a question whether or not the space $\mathcal{B}_1^b(H(U))$ satisfies the "barycentric formula" or it is uniformly closed. Solving these problems we use in an essential way the fine topology methods and Choquet's theory of simplicial spaces.

The situation is quite different when replacing the function space $H(U)$ by the space of continuous affine functions on a compact convex set and $\mathcal{B}_1^b(H(U))$ by the space of Baire–one affine functions (positive theorems of Choquet and Mokobodzki).

The exposition will be quite elementary and all basic notions will be explained.

Random planar graphs

Marc Noy[1]

Universitat Politècnica de Catalunya

Let G be a graph chosen uniformly at random among all labelled planar graphs with n vertices. Which are the typical properties of such a random planar graph?

We show that G has about Cn edges, where $C = 2.21326\ldots$ is a constant completely determined, and that deviations from Cn are with high probability of small order. Among other properties, we also show that G is connected with probability tending to a constant $0.96325\ldots$

The basic technique we use in the proofs is singularity analysis of counting generating functions, considered as complex valued functions.

Can quantum theory help us prove theorems?

Pavel Pudlák

Czech Academy of Sciences, Prague

It seems very probable that quantum computers could solve some problems (e.g. factoring of integers) much faster than any classical computers. A natural question is whether quantum computers could also help us produce proofs that would be too long or too difficult to find by classical means. Though this problem is well known, no results have been obtained so far. We shall study this problem from the point of view of proof complexity. Our aim is to find the quantum concept that corresponds to the classical Frege proof systems.

[1] joint work with Omer Gimenez

Section Computational Statistics and Data Analysis

Use of Permutation Principle in detection of structural changes in regression

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Keywords: Linear regression, structural changes, L_2-procedures, permutation principle, Monte Carlo, change-point problem.

We consider the regression model with a change after an unknown time point m_n, i.e.

$$Y_{in} = \boldsymbol{x}_{in}^T\boldsymbol{\beta} + \boldsymbol{x}_{in}^T\boldsymbol{\delta}_n \cdot I\{i > m_n\} + e_i, \quad i = 1\ldots, n, \tag{1}$$

where $m_n\,(\leq n)$, $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_p)^T$ and $\boldsymbol{\delta}_n = (\delta_{1n}, \ldots, \delta_{pn})^T \neq \mathbf{0}$ are unknown parameters, $\boldsymbol{x}_{in} = (x_{i1,n}, \ldots, x_{ip,n})^T$, $x_{i1,n} = 1$, $i = 1, \ldots, n$, are known design points and $e_1, \ldots, e_n$ are iid random errors fulfilling regularity conditions specified below. Function $I\{A\}$ denotes the indicator of the set A.

Model (1) describes the situation where the first m_n observations follow the linear model with the parameter $\boldsymbol{\beta}$ and the remaining $n - m_n$ observations follow the linear regression model with the parameter $\boldsymbol{\beta} + \boldsymbol{\delta}_n$. The parameter m_n is usually called the *change point*.

In the lecture we focus on the testing problem:

$$H_0 : m_n = n \qquad \text{against} \qquad H_1 : m_n < n. \tag{2}$$

Approximations to the critical values needed for this testing problem can be obtained through the limit distribution of the respective test statistics under H_0, however, such approximations are usually not satisfactory. Therefore, we proposed another possibility, namely, the approximations based on the application of the permutational principle, of course, suitably modified for the situation of regression models. We will discuss this approach in the lecture and give some examples based on real data illustrating its advantages and disadvantages.

Cluster analysis in Phylogeny

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Keyword: Clustering, philogeny, evolutionary tree

Cluster analysis is used in genetics to generate hypotheses of organisms evolution. It is namely the hierachical methods producing dendrograms interpreted as evolutionary trees that are are applied to asses genetical similarity of biological objects.

The contribution concerns the influence of clustering parametres (i.e. encoding the nucleotides, distances, clustering method) to the configuration of resulting trees.

Finally, the other potentionality of cluster analysis – classification – is presented. There are studied appropriate criteria of partitioning the observation into groups.

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Competing neural networks as models for non stationary financial time series

Joseph Tadjuidje Kamgaing

visiting Charles University of Prague

We consider time series switching between different dynamics or phases, e.g. a generalized mixture of first order nonlinear AR-ARCH models with two dynamics

$$X_t = S_t(m1(X_{t-1}) + \sigma_1(X_{t-1})\epsilon_t) + (1 - S_t)(m_2(X_{t-1}) + \sigma_2(X_{t-1})\epsilon_t)$$

The hidden process S_t is a first order Markov chain with values in $\{0, 1\}$, the residuals ϵ_t ot are i.i.d. with mean 0 and variance 1, the autoregressive, m_1 and m_2, and volatility, σ_1 and σ_2, functions are unknown. We first present some conditions that ensure the asymptotic stability(Geometric Ergodicity) of the process and define a version of the likelihood function under mild assumptions. Further, based on the likelihood we investigate the behavior of feedfoward networks for estimating the autoregressive and volatility functions and identifying the changepoints between different phases.

Since the process S_t is not observable, we design a version of the Expectation Maximization algorithm that account for solving the problem numerically. In fact, this algorithm consists of assuming in the Expectation step that the parameters of the networks functions are known and to estimate the $\widehat{S}_t$. Considering now the $\widehat{S}_t$ the parameters of the networks functions are derived in theMaximization step. Both steps are iterated until a stopping criterion is satisfied.

Based on these estimations, we construct a trading strategy that we apply on real life data and compare the results with those of the classical Buy and Hold strategy.

On ordering of splits and Gray code (a new trick and its long history)

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In COMPSTAT'98 article [4], Klaschka and Mola dealt with an economical way of calculating, in the tree-based methods, all the $2^{n-1} - 1$ values of splitting criterion for the splits based on an n-valued categorical variable. We proposed a specific ordering of splits suitable for recalculating one value from another.

The core of the paper was a combinatorial idea: The $(n-1)$-tuples of 0's and 1's coding the splits can be ordered in a sequence so that any two successive elements differ in exactly one position.

Only later I learned that the same combinatorial idea had already been utilized as early as in the sixties in the all possible subsets regression, see [1], [6].

Moreover, even in the sixties the idea was not quite novel: It was an independent reinvention of so called Gray code, for which Frank Gray took out a patent in 1953 (see [2], [5]).

Gray code seems to possess a high potential of being repeatedly reinvented. By the way, Gray was not the first to invent it: According to [3], it was used by Emile Baudot's telegraph, awarded a gold medal at the Universal Exposition in Paris in 1878.

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Variation and information closures of exponential families

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The variation distance closure of an exponential family with a convex set of canonical parameters is described, not assuming any regularity conditions. The description relies on the concept of convex core of a measure and its faces. The closure is a subset of an extension of the exponential family, defined as a union of exponential families over the faces. The crucial new ingedient is a concept of accessible faces of a convex set. The closures in reversed information divergence are expressed via the variation closures of auxiliary subfamilies. Also, variation convergence and information convergences in the extension are characterized.

Non linear models in financial time series

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Modelling financial time series is a field in constant growth, as much from a theoretical point of view as from estimation algorithms development. Under linearity hypothesis, Box

[2]joint work with Imre Csiszár
[3]joint work with M. D. Márquez

and Jenkins methodology for ARIMA models, provides a well developed statistical theory and computational tools which are readily available. But financial time series exhibit some features that cannot be described by linear time series models; this situation has motivated many authors to consider non linear alternatives. In this lecture we present the distributional and empirical properties of the most commonly used non linear models in variance (ARCH, GARCH and SV), non linear models in mean (Bilinear and TAR) and the new developments that allow to capture both forms of non linearity. Finally, applications to real data sets are presented.

Regression models of animal growth

Helena Nešetřilová

Czech University of Agriculture

Classical growth models are nonlinear regression models with three or four parameters. Due to nonlinearity, statistical properties of such models can be studied only in relation to concrete data sets. Some of these properties will be discussed for data on growing bulls. In some cases, complex nature of growth can be more adequately modelled by superposition of two (or more) growth cycles.

Fisher linear discriminant analysis as a method for protein classification

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Keyword: Fisher linear discriminant analysis, protein classification, prediction of microarrays

Statistical decision rules can be used in genetics, more accurately in microarray, to classify proteins into several classes according information taken from other measured variables.

The contribution concerns on using Fisher linear discriminant analysis on a real data consisting of 268 proteins classified according 400 variables into 42 classes. The classification task of such a case is typical for microarray where number of variables is higher than number of observed objects (also known as $p \gg n$ problem). The data was investigated several times earlier in e. g. [1], [2] and [3]; the project web page of these studies as well as with other results is `http://www.dkfz.de/biostatistics/protein/DEF.html`. A lot of linear and nonlinear methods was used, the lowest error was reached with nonlinear method called Support Vector Machines.

It will be shown that with a good implementation of Fisher discriminants the linear method can beat nonlinear ones. A small comparison with classical implementation by `lda()` function in `R` (`http://www.r-project.org`) will be shown as well.

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TV audience measuring

Jaroslav Štěrba

TNS Audiencia de Medios, Barcelona

This talk will deal with statistical methods suitable for monitoring of the TV watching. These methodologies comprise not only the statistical methods connected to survey samples but also a bunch of side conditions connected to psychological and sociological aspects of the problem. During the lecture I will concentrate of the question how to balance all these requirements, often going one against the other.

GMM weighted estimation

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The point estimation was from the very beginning of the statistics (and econometrics) one of the key topics. In the early days, the unbiasedness was assumed to play the crucial role but later the *(weak) consistency* overtook the governance.

The (classical and/or robust) statistics developed a bunch of "principles", heuristics of which promised to yield the estimators being not only consistent but also premiant in a widely considered competition (achieving e. g. efficiency). Some of them worked, some not. Typically, the estimator was given as a solution of a (vector) equation (*normal equations*) - interpretable as p-tuple of orthogonality conditions (of residuals to the columns of design matrix, e. g.).

The consistency requires orthogonality of residuals to the (estimated) model and hence the estimators are defined as solution of a q-tuple of orthogonality conditions ($p \leq q$). It allows for direct employment of additional information about the parameter in question. In such a case we speak about *Generalized Method of Moments estimation*.

Despite the forty years of robust studies, the econometrics haven't taken seriously (possible) fatal consequences of a slight deviation of the assumed model from the underlying one as Fisher did already in 1922 or of a few contaminating observations as considered by Hampel or Huber much later on.

Since the weighting down the order statistics of squared residuals appeared to be powerful tool for influential-points-recognition, the present paper offers an idea of the *generalized method of moments weighted estimators* and shows that *the least weighted squares* are special case of them.

Section Discrete Mathematics and Combinatorics

Improved lower bound for the vertex-connectivity of $(\delta; g)$-cages

Camino Balbuena

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A (δ,g)-cage is a δ-regular graph with girth g and with the least possible number of vertices. It has been proved that every $(\delta; g)$-cage with $\delta \geq 3$ is 3-connected [See the papers by M. Daven and C. A. Rodger, $(k; g)$-cages are 3-connected, *Discrete Math.* 199 (1999), 207–215; or the paper by T. Jiang and D. Mubayi, Connectivity and separating sets of cages, *Journal of Graph Theory* 29 (1998), 35–44].

In this work we prove that all (δ,g)-cages are r-connected with $r \geq \sqrt{\delta+1}$ for $g \geq 7$ odd. This result supports the conjecture of Fu, Huang and Rodger that all $(\delta; g)$-cages are δ-connected.

A new approach to finite semifields

Simeon Ball

Universitat Politècnica de Catalunya

A *finite semifield* is a finite set S with two operations, addition and multiplication, such that $(S, +, \circ)$ satisfies all the axioms of a field except (possibly) associativity of multiplication.

A semifield can be used to coordinatise a projective plane of order $|S|$ and we are interested in finding semifields that produce non-isomorphic projective planes. Two semifields are said to be *isotopic* if they coordinatise isomorphic planes. There are less than roughly 20 known families of (mutually non-isotopic) semifields. Unless it is immediate that two semifields are not isotopic it is generally difficult to establish whether or not they are. Above all, the goal in this area is to construct many more families of non-isotopic semifields. The first semifields were discovered by Dickson, roughly 100 years ago with more examples given later by Albert (1950's), Knuth (1960's), Cohen and Ganley (1980's) and various families due to Kantor, amongst others, have been constructed in the last twenty years.

If S is finite then it can be shown that $|S| = q^n$ for some prime power q and that S can viewed as a vector space of rank n over $\mathbb{F}_q$, where multiplication is given by $a_{ijk} \in \mathbb{F}_q$ by the rule

$$e_i \circ e_j = \sum_{k=1}^{n} a_{ijk} e_k,$$

where $\{e_1, e_2, \ldots, e_n\}$ is a basis for S over $\mathbb{F}$. Knuth was first to note that any permutation of the subscripts produces another semifield, so there are six semifields associated with any semifield.

In this talk I shall present a new way to construct finite semifields of order q^n from two subspaces of a vector space of rank rn over $\mathbb{F}_q$, for some r. In fact any finite semifield can be constructed in this way for some $r \leq n$, moreover any known semifield of order q^n can be constructed from two subspaces of a vector space of rank $2n$ or rank $3n$ over $\mathbb{F}_q$.

The construction also provides us with a new operation (not one of the six due to Knuth) which produces more semifields in the case when $r = 2$.

A polynomial power-compositions determinant

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Let n and p be positive integers. A *p-composition* of n is a p-tuple of non-negative integers $\alpha = (\alpha_1, \ldots, \alpha_p)$ such that $\alpha_1 + \cdots + \alpha_p = n$. Denote by $C(n,p)$ the set of p-compositions of n. If $\alpha = (\alpha_1, \ldots, \alpha_p)$ and $\beta = (\beta_1, \ldots, \beta_p)$ are p-compositions of n, we denote $\alpha^\beta = {\alpha_1}^{\beta_1} \cdots {\alpha_p}^{\beta_p}$, where to be consistent, it is assumed that $0^0 = 1$. The power-compositions determinant is the determinant

$$\Delta(n,p) = \det_{\alpha,\beta \in C(n,p)} (\alpha^\beta).$$

The value of $\Delta(n,p)$ is given in [1]:

$$\Delta(n,p) = \prod_{k=1}^{\min\{n,p\}} \left(n^{\binom{n-1}{k}} \prod_{i=1}^{n-k+1} i^{(n-i+1)\binom{n-i-1}{k-2}} \right)^{\binom{p}{k}}.$$

[4]joint work with Antonio Montes

Recently, C. Krattenthaler in the complement [3] to its impressive *Advanced Determinant Calculus*[2], has given an equivalent formula for $\Delta(n,p)$ and has stated the following conjecture supported by computer experiments:

$$\det_{\alpha,\beta\in C(n,p)}((x+\alpha)^{\beta}) = (px+n)^{\binom{n+p-1}{p}}\prod_{i=1}^{n} i^{(n-i+1)\binom{n+p-i-1}{p-2}},$$

where x is a variable and $x+\alpha$ is short for $(x+\alpha_1,\ldots,x+\alpha_p)$. In this talk we prove this conjecture using a method that can be useful for other combinatorial determinants.

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Constraint satisfaction problems and expressiveness

Victor Dalmau

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Constraint satisfaction problems arise in a wide variety of domains, such as combinatorics, logic, and artificial intelligence. In recent years, much effort has been directed towards classifying the complexity of all families of constraint satisfaction problems. It has been shown that it is possible to associate to every subclass of the CSP an algebra **A** such that the complexity the corresponding CSP is completely determined by the properties of **A**.

Let **A** be any algebra over a finite universe. We define the *expressiveness* of **A** as the mapping $\mathrm{ex}_{\mathbf{A}} : \mathcal{N} \to \mathcal{N}$ that given a natural number n returns the number subuniverses of $\mathbf{A}^n$. Some algebras **A** that lead to tractable cases of the CSP such as Mal'tsev algebras, or algebras containing a near-unanimity operations among their term operations, satisfy the following property: $\log \mathrm{ex}_{\mathbf{A}}(n) \leq p(n)$, for some polynomial p. We call any such algebra *polynomially expressible.* We conjecture that all polynomially expressible algebras give rise to families of constraint satisfaction problems solvable in polynomial time. In this talk we shall present some initial results in this direction.

List-colouring squares of sparse subcubic graphs

Zdeněk Dvořák

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Colouring the square of a graph naturally arises in connection with the distance labelings, which have been studied intensively. We consider the problem for sparse subcubic graphs. We show that the choosability $\chi_\ell(G^2)$ of the square of a subcubic graph G of maximum average degree d is at most four if $d < 24/11$ and G does not contain a 5-cycle, $\chi_\ell(G^2)$ is at most five if $d < 7/3$ and $\chi_\ell(G^2)$ at most six if $d < 5/2$. Wegner's conjectures claims that the chromatic number of the square of a subcubic planar graph is at most seven. Our result implies that $\chi_\ell(G^2)$ is at most four if $g \geq 24$, it is at most 5 if $g \geq 14$ and it is at most 6 if $g \geq 10$. For lower bounds, we find a planar subcubic graph G_1 of girth 9 such that $\chi(G_1^2) = 5$ and a planar subcubic graph G_2 of girth five such that $\chi(G_2^2) = 6$. As a consequence, we show that the problem of 4-colouring of the square of a subcubic planar graph of girth $g = 9$ is NP-complete.

Graph orders arising from locally constrained homomorphisms

Jiří Fiala[5]

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Degree refinement matrices have tight connections to graph homomorphisms that locally, on the neighborhoods of a vertex and its image, are constrained to three types: bijective, injective or surjective. If graph G has a homomorphism of given type to graph H, then we say that the degree refinement matrix of G is smaller than that of H. This way we obtain three partial orders. We present algorithms that will determine whether two matrices are comparable in these orders. For the bijective constraint no two distinct matrices are comparable. For the injective constraint we give a PSPACE algorithm, which we also apply to disprove a conjecture on the equivalence between the matrix orders and universal cover inclusion.

[5] joint work with Daniël Paulusma and Jan Arne Telle

Computing the Tutte polynomial on graphs of bounded clique-width

Peter Hliněný[6]

CS - FEI VŠB Ostrava, CZ

The Tutte polynomial is a notoriously hard graph invariant, and efficient algorithms for it are known only for a few special graph classes, like for those of bounded tree-width. The notion of clique-width extends the definition of cograhs (graphs without induced P_4), and it is a more general notion than that of tree-width. We show a subexponential algorithm (running in time $\exp O(n^{1-\varepsilon})$) for computing the Tutte polynomial on graphs of bounded clique-width. In fact, our algorithm computes the so-called U polynomial of such a graph.

The Middle levels problem

Tomáš Kaiser[7]

University of West Bohemia, Plzeň

Let B_k be the bipartite graph formed by the middle two levels of the lattice of subsets of a $(2k+1)$-element set. The well-known Middle levels problem is the question whether B_k is hamiltonian for all $k \geq 1$. As a step towards an answer, we show that B_k has a spanning 3-connected cubic subgraph.

[6] joint work with Omer Gimenez and Marc Noy

[7] joint work with P. Horák, M. Rosenfeld and Z. Ryjáček

On growth rates of closed sets of permutations, set partitions, ordered graphs and other objects

Martin Klazar

Charles University, Prague

If X is a closed set of finite permutations (i.e., a lower ideal in $(\mathcal{S}, \preceq)$ where $\mathcal{S}$ is the set of all finite permutations and $\preceq$ is the standard containment ordering) and $X_n \subset X$ denotes the set of all n-permutations in X, then the counting function $n \mapsto |X_n|$ is subject to various dichotomies and restrictions forbidding many functions to have this form; this was shown (*Electronic J. of Combinatorics*, 2003) by T. Kaiser and M. Klazar. For example, either $|X_n| \geq n$ for all $n \geq 1$ or $|X_n|$ is eventually constant, or—another dichotomy—either $|X_n| \leq n^c$ for all $n \geq 1$ with a constant $c > 0$ or $|X_n| \geq F_n$ for all $n \geq 1$, where $F_n = 1, 2, 3, 5, 8, 13, \ldots$ are the Fibonacci numbers.

In my talk I will present generalizations and extensions of these results to other classes of objects (like those mentioned in the title) and other containments, and I will discuss a general approach to obtain them uniformly as instances of a general metaresult.

Group colorings of graphs

Daniel Kráľ[8]

MFF UK Praha and TU Berlin

We present several results on group coloring introduced by Jaeger et al. Group coloring is the dual concept of group connectivity, non-homogenous variant of nowhere-zero flows. We show that the group chromatic number of a graph with minimum degree d is greater than $d/(2 \log d)$ and answer several open questions on the group chromatic number of planar graphs. We also establish that the decision problem whether a graph G is A-colorable is Π_2-complete for every fixed Abelian group A of order three or more.

[8]joint work with P. Nejedlý, O. Pangrác and H.-J. Voss

4-edge-coloured graphs and 3-manifolds

Roman Nedela

Matej Bel University
Banská Bystrica, Slovakia

The homeomorphism problem on 3-manifolds attracts mathematicians since the beginnings of 20-th century. A particular but fundamental instance of it is known as the Poincare conjecture. Its central position in the core of classical mathematics is stressed by the fact that a formulation of the problem belongs to the list of the seven millennium problem proclaimed by the Clay Institute of Mathematics. The aim of the talk is to show how some classical mathematical problems can be formulated in a combinatorial way. One of the advantages of this approach is a possibility to use computer to study 'small' examples. The most developed part of a combinatorial approach is knot theory based on a well-known fact that any 3-manifold can be constructed as a branched cover over the 3-sphere, where the set of branch points forms a knot. In our talk we present another, not so known combinatorial approach to 3-manifolds. We first introduce a combinatorial counterpart of the classical homeomorphism problem on 3-manifolds using a theory built by Pezzana and his successors. Pezzana proved that every closed compact orientable 3-manifold $\mathcal{M}$ can be represented by a bipartite 4-edge-coloured 4-valent graph called a crystallisation of $\mathcal{M}$. Crystallisation is a 4-valent 4-edge-coloured graph such that removal of any monochromatic (perfect) matching yields a planar graph. This way 3-dimensional objects are replaced by 1-dimensional which allows us to employ combinatorial methods to study the homeomorphism problem. We explain a result of Casali and Grasselli showing that 3-manifolds of Heegaard genus g can be represented by crystallisations with a very simple structure which can be described by a $2(g+1)$-tuple of non-negative integers. The sum of first $g+1$ integers is called complexity of the admissible $2(g+1)$-tuple. If c is the complexity then the number of vertices of the associated graph is $2c$. We give a combinatorial definition of the Heegaard genus and show how to classify 3-manifolds of Heegard at most one using graph theoretical results. Then we introduce an algorithm to derive a presentation of the fundamental group of a 3-manifold from the representation by means of a $2(g+1)$-tuple of integers. A 3-manifold is called prime if it cannot be expressed as a connected sum of non-trivial 3-manifolds. We show how to classify all prime 3-manifolds of Heegaard genus 2 described by 6-tuples of complexity at most 21. Finally, we add some remarks on the celebrated Thurston's geometrisation conjecture and on recent progress in the topic.

Distance graphs with maximum chromatic number

Oriol Serra[9]

Universitat Politècnica de Catalunya

Let D be a finite set of integers. The distance graph $G(D)$ has the set of integers as vertices and two vertices at distance $d \in D$ are adjacent in $G(D)$. A conjecture of Xuding Zhu states that if the chromatic number of $G(D)$ achieves its maximum value $|D|+1$ then the graph has a clique of order $|D|$. We prove that the chromatic number of a distance graph with $D = \{a, b, c, d\}$ is five if and only if either $D = \{1, 2, 3, 4k\}$ or $D = \{a, b, a+b, a+2b\}$. This confirms Zhu's conjecture for $|D| = 4$.

The mean Dehn function of abelian groups

Enric Ventura[10]

Universitat Politècnica de Catalunya

In this talk we will consider Dehn functions of groups. After a quick review of their interest and main properties, we'll introduce the much more modern notion of "mean Dehn function". In the second part of the talk, we'll scketch the proof of the following theorem: "the mean Dehn function of a finitely generated abelian group is $O(n(\log n)^2)$". This result makes a big contrast with the well known facts that the Dehn function of abelian groups is quadratic, and that there is no group with Dehn function between quadratic and linear. The proof consists on a detailed combinatorial analysis involving several countings of paths of given length in the integral lattice Z^n.

[9] joint work with Javier Barajas

[10] joint work with Oleg Bogopolski

Section Homotopy Theory

Zeta functions and enumerative problems over finite fields

Tibor Beke

Hungarian Academy of Sciences, Budapest

Suppose one has an enumerative combinatorial problem that can be evaluated over the finite fields with q, q^2, q^3, ... q^k elements, giving rise to the sequence of counts N_1, N_2, N_3, ... N_k. When is the associated generating function

$$g(t) = \sum_{k=1}^{\infty} N_k t^k$$

a rational function?

Part of the Weil conjectures (ie the theorem of Grothendieck-Deligne) is that if one counts the number of common zeroes of a set of polynomials over bigger and bigger finite fields, then the associated generating function is rational. We review the cohomological proof and subsequent extensions: to counting problems that involve first-order quantifiers (due to Kiefe, Macintyre and others) and field extensions (due to Wan). I continue with my own work, and mention some open problems.

Simplicial orthogonality

Carles Casacuberta

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An object X and a morphism f in a category $\mathbf{C}$ are called orthogonal if the arrow $\mathbf{C}(f, X)$ is bijective. In the framework of simplicial model categories, the notion of simplicially enriched orthogonality plays an important role: X and f are said to be simplicially orthogonal if the arrow $\mathrm{map}(f, X)$ is a weak equivalence of simplicial sets. We will explain why the condition that a given class of objects D and a given class of morphisms S be the simplicial orthogonal complement of each other is necessary and sufficient to ensure the existence of a homotopy localization functor L such that D is the class of L-local objects and S is the class of L-equivalences, under suitable assumptions on the model category and possibly using large-cardinal axioms.

Localization of algebras over operads

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We prove that homotopical localization functors in topological symmetric monoidal categories preserve algebras over cofibrant topological operads. It follows, among other consequences, that stable homotopical localizations commuting with suspension preserve A-infinity and E-infinity algebras.

What is homotopy theory good for?

M. Markl

We will discuss some recent applications of homotopy theory and homological algebra in mathematical physics, namely Merkulov's proof of the Deformation Quantization Theorem by Kontsevich.

Triangulated categories and translation cohomology

Fernando Muro

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In this talk we will introduce the translation cohomology of a pair given by an additive category $\mathbf{A}$ and a self equivalence $t\colon \mathbf{A} \to \mathbf{A}$. We will show how, under two simple axioms, a 3-dimensional translation cohomology class $\nabla \in H^3(\mathbf{A}, t)$ induces a triangulated structure on $\mathbf{A}$ with translation functor t. All triangulated categories coming from a stable homotopy theory arise in this way.

Are all localizing subcategories of a stable homotopy category coreflexive?

J. Rosický[11]

Masaryk University, Brno

Localizing subcategories are very important in stable homotopy theory and there is not known any example of a localizing subcategory $\mathcal{L}$ without a localization functor, which is the same thing as $\mathcal{L}$ not being coreflective. We will confirm the suspicion of M. Hovey, J. H. Palmieri and N. P. Strickland that the answer may depend on set theory by showing that, assuming Vopěnka's principle, every localizing subcategory $\mathcal{L}$ of the homotopy category $\mathcal{S}$ of spectra is coreflective. Moreover, $\mathcal{L}$ is generated by a single object and, dually, every colocalizing subcategory $\mathcal{C}$ of $\mathcal{S}$ is reflective and generated by a single object. The consequence is that every localizing subcategory of $\mathcal{S}$ is a cohomological Bousfield class.

[11] joint work with C. Casacuberta and J. Gutiérrez

Space of maps from the Hawaiian earring to a CW complex

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We discuss the homotopy type of the space of continuous functions from the Hawaiian earring to a CW complex, endowed with the compact open topology.

Section Logic

Set theory and reals

Bohuslav Balcar

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In set theory, a real number is a synonymum for a subset of natural numbers. We shall deal with combinatorial properties of the partial order of the set of all infinite subsets of natural numbers ordered by inclusion. Motivation comes from the forcing theory.

Speed-up theorem and Kreisel's Conjecture

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Gödel in 1936 announced a speed-up theorem (GST) that holds when one switches from a weaker formal system for arithmetic to a stronger one. Theorems with long proofs in a formal system T can get much shorter proofs in a formal system S. Gödel did not give a proof of his result but subsequently proofs were given by Parikh, by Krajicek and in a general version by Buss. Kreisel's Conjecture (KC) is an open problem in the study of lengths of proofs. KC states in the following way: If there exists k such that the formal system for Peano Arithmetic PA proves $A(s^{(n)}(0))$ in k steps for every n, then PA proves $(\forall x)A(x)$. The idea behind the conjecture is that a short proof of an instance $A(s^{(n)}(0))$ of $A(x)$ for large n cannot use the whole information about the numeral for n and thus should generalize to other numbers too.

We briefly recapitulate some basic fact about KC, GST and we show that the following proposition

Proposition: There exists a formal system T such that T has an unbounded speed-up over PA but is still conservative extension of PA.

is a corollary of KC but we also give a proof not using any unproven assumption.

Weak predicate fuzzy logics

Petr Cintula

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The class of weakly implicative fuzzy logic (WIFL) was introduced to encompass the existing class of the so-called fuzzy logics in one general framework. Roughly speaking, WIFL is the class of propositional logics (understood as a consequence relation) complete w.r.t. linearly ordered logical matrices of truth values.

In this talk we concentrate on first-order variant of logics in WIFL. In particular we deal with problems appearing in their very basic aspects. As WIFL is very broad class we run into problems with proper formulating of an axiomatic system of these predicate logics and then with proving the completeness theorem.

The former one lies in the fact that the connective of max-disjunction plays a crucial role in the predicate fuzzy logics known from the literature. However, in WIFL there are logics without this connective in the language. The later one is caused by fact that by the transition to the first-order we lose (in some cases) some important proof-theoretic meta-theorems of the propositional logics necessary for the completeness proof.

We try to overcome both problems and provide some partial results. Generally, we can say that for good-behaving propositional logics (with some form of deduction theorem, definable lattice structure, etc.) the transition to the first-order is rather smooth and painless and so we concentrate on those problematic ones.

New forms of the Deduction Theorem and Modus Ponens

Josep M. Font[12]

University of Barcelona

This paper studies, with techniques of Abstract Algebraic Logic, the effects of putting a bound on the cardinality of the set of side formulas in the Deduction Theorem, viewed as a Gentzen-style rule, and of adding additional assumptions inside the formulas present in Modus Ponens, viewed as a Hilbert-style rule. As a result, a denumerable collection of new Gentzen systems and two new sentential logics have been identified. These logics

[12]joint work with Félix Bou and José-Luis García Lapresta

are weaker than the positive implicative logic. We have determined their algebraic models and the relationships between them, and have classified them according to several standard criteria of Abstract Algebraic Logic. One of the logics is protoalgebraic but neither equivalential nor weakly algebraizable, a rare situation where very few natural examples were hitherto known. In passing we have found new, alternative presentations of positive implicative logic, both in Hilbert style and in Gentzen style, and have characterized it in terms of the restricted Deduction Theorem: it is the weakest logic satisfying Modus Ponens and the Deduction Theorem restricted to at most 2 side formulas. The algebraic part of the work has lead to the class of quasi-Hilbert algebras, a quasi-variety of implicative algebras introduced by Pla and Verdú in 1980, which is larger than the variety of Hilbert algebras. Its algebraic properties reflect those of the corresponding logics and Gentzen systems.

On an infinite-valued Lukasiewicz logic that preserves degrees of truth

Angel J. Gil[13]

Universitat Pompeu Fabra, Barcelona

Infinite-valued Lukasiewicz can be defined from a class of matrices constituted by Wajsberg algebras (also called MV-algebras) where the set of designated values is an arbitrary implicative filter. This logic is protoalgebraic, algebraizable but not selfextensional. It does not satisfy the Deduction-Detachment Theorem, nor the Graded Deduction Theorem, but it satisfies the Local Deduction-Detachment Theorem.

In this paper we will study a new logic, determined also by Wajsberg algebras, but focusing on the order relation instead of the implication. This new logic is an example of a "logic that preserves degrees of truth", and has the following properties: it can be defined from the class of matrices constituted by Wajsberg algebras where the set of designated values is an arbitrary lattice filter, it is not protoalgebraic, not algebraizable and selfextensional. It does not satisfy the Deduction-Detachment Theorem, nor the Local Deduction-Detachment Theorem, but it does satisfy the Graded Deduction Theorem.

Since the new logic is selfextensional and has conjuntion, it has a Gentzen system that is both fully adequate for it and algebraizable, having the same algebraic counterpart as the logic. Wegive a a sequent calculus of the Gentzen system corresponding to this logic, we study its properties and models, and we determine several relationships between the Gentzen system and the infinite valued Lukasiewicz logic.

[13]joint work with Josep Maria Font, Antoni Torrens and Ventura Verdú

A refined isomorphism theorem in categorical abstract algebraic logic

José Gil-Férez

University of Barcelona

In [1] a characterization of algebraizability of a sentential logic $\mathcal{S}$ is obtained in terms of the existence of an isomorphism between the lattice of theories of $\mathcal{S}$ and the lattice of theories of the equational consequence of its equivalent algebraic semantics $\mathbf{K}$, commuting with substitutions. In [2] we find a generalization of this result in Teorem V.3.5, stating that two deductive systems are equivalent if, and only if, there exists an isomorphism between their lattices of theories, commuting with substitutions. In turn, in [5] a new generalization of this result is exhibited for Gentzen systems: Theorem 2.19.

Each of these systems is a generalization of the previous one: sentential logics and equational consequences of a class of algebras are particular cases of k-deductive systems, which are particular cases of Gentzen systems. But there are other kinds of deductive systems with a similar Isomorphism Theorem that are not, nor extend, Gentzen systems.

A generalization of all these systems are π-institutions, which were introduced for the first time by Fiadeiro and Sernadas in [3], inspired by the work on institutions of Goguen and Burstall in [4]. Institutions cover all these deductive systems and also formalize other multi-sorted ones coming form computing science. π-institutions in turn focus attention in the syntax instead of in semantics, as do institutions. For both, a categorical context organizing the information is required.

In [6] Voutsadakis proved the following result:

Theorem (Voutsadakis). If $\mathcal{I}$ and $\mathcal{I}'$ are two term π-institutions, then they are deductively equivalent if, and only if, there exists an adjoint equivalence $\langle F, G, \eta, \varepsilon \rangle : \mathbf{Th}\mathcal{I} \rightharpoonup \mathbf{Th}\mathcal{I}'$ that commutes with substitutions.

This result is a generalization of Theorem V.3.5 of [2]. However, in spite of its abstraction level, it is not a generalization of Theorem 2.19 of [5], since not all Gentzen systems can be exhibited (in fact, only those that are k-systems can) as term π-institutions. This shows that this condition is probably too strong. But it cannot be just removed: I will provide two π-institutions which are not deductively equivalent but with isomorphic categories of theories (through an isomorphism commuting with substitutions).

The objective of eliminating absolutely the conditions over the π-institutions in the characterization theorem of deductive equivalence is then vane. However, I will offer a way of extending the result which will cover Gentzen systems among others. To do this, the notion of Grothendieck construction of a **Cat**-valued functor will be used. We obtain then the following extended version of the Isomorphism Theorem:

Theorem. If $\mathcal{I}$ and $\mathcal{I}'$ are two multi-term π-institutions, then they are deductively equivalent if, and only if, there exists an adjoint equivalence $\langle F, G, \eta, \varepsilon \rangle : \mathbf{Th}\mathcal{I} \rightharpoonup \mathbf{Th}\mathcal{I}'$ that commutes with substitutions.

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Axiomatic extensions of Monoidal Logic

Joan Gispert

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In this communication we will present an algebraic study of some axiomatic extensions of the Monoidal Logic, also named FLew (Full Lambda Calculus plus exchange and weakening). In particular we will deal withaxiomatic extensions that satisfy the pre-linearity condition and wil study their associated algebras: MTL-algebras. Not much is known on how to construct MTL-algebras. Althouguht MTL-algebras are very close to BL-algebras, they lack to have a representation theorem like the representation as ordinal sum. Jenei presented some contructions for the case of involutive MTL- algebras. We associate to each construction an equational class, we study its associated logic, we answer the question of having or not standard completeness. Moreover we generalise those classes to the non involutive case.

Completeness results for product logic with truth-constants

Lluís Godo[14]

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In this paper we investigate expansions of Product fuzzy logic by adding into the language a countable set of truth-constants and by adding the corresponding book-keeping axioms for the truth-constants. We study algebraic semantics and prove standard completeness for all these expansions. We also investigate the issue of finite strong standard completeness. We conclude with some remarks on the expansion with truth-constants of other schematic extensions of BL and MTL.

Some model theory in fuzzy logic

Petr Hájek

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A model-theoretic characterization of conservative extensions of axiomatic theories over the basic fuzzy predicate logic BL will be presented. Furthermore, the notion of witnessed models will be discussed, a model being witnessed if for each quantified formula its truth value in the model (defined as the supremum/infimum of values of instances) is in fact maximum/supremum. A completeness theorem for witnessed semantics will be stated.

[14]joint work with P. Savicky, R. Cignoli, F. Esteva and C. Noguera

Bounded distributive lattices with strict implication

Ramon Jansana

We will introduce the variety of weakly Heyting algebras. They are bounded distributive lattices with an implication that has the properties of the strict implication of the normal modal logic K. This variety has as subvarieties the varieties of Heyting algebras, Basic algebras of Visser's logic, and all the varieties that correspond to the strict implication fragments of the normal modal logics. We will present a Priestly style duality for weakly Heyting algebras and several results that show that the study of the lattice of varieties of weakly Heyting algebras encompasses both the stydy of the lattice of varieties of normal modal algebras and the study of the lattice of varieties of Heyting algebras.

PA sets, 1-random sets and Π^0_1 classes

Antonín Kučera

Charles University, Prague

A survey of results and some open problems concerning algorithmic randomness and complete extensions of PA will be presented. The structure of T-degrees of 1-random sets and the properties of so-called K-trivials will be discussed.

Section Real and Functional Analysis

Multiple fractional integral with Hurst parameter lesser than 1/2

Xavier Bardina

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We construct a multiple Stratonovich-type integral with respect to the fractional Brownian motion with Hurst parameter $H < 1/2$. This integral is obtained by a limit of Riemann sums procedure in the SolĆ and Utzet sense. We also define the suitable traces to obtain the Hu-Meyer Formula that gives the Stratonovich integral as sum of ItŞ integrals of these traces. Our approach is intrinsic in the sense that we do not make use of integral representation of the fractional Brownian motion in terms of the ordinary Brownian motion.

Factorization of a class of almost-periodic triangular symbols and related Riemann-Hilbert problems

Cristina Camara

Universidade Técnica de Lisboa

The Fredholmness and invertibility of finite-interval convolution operators acting on spaces $L_2(I)$, where I is a compact interval in $\mathbb{R}$, is closely related to the Wiener-Hopf factorization of its matrix-valued symbol. This factorization is studied for some classes of symbols G whose entries are almost-periodic polynomials with Fourier spectrum in the group $\alpha\mathbb{Z} + \beta\mathbb{Z} + \mathbb{Z}$ $\alpha, \beta \in]0,1[$, $\frac{\alpha}{\beta} \notin \mathbb{Q}$). The factorization problem is solved by calculating one solution to the Riemann-Hilbert problem $G\Phi_+ = \Phi_-$ in $L_\infty(\mathbb{R})$ and obtaining a second linearly independent solution by means of an appropriate transformation on the space of solutions of the Riemann-Hilbert problem. Some unexpected, but interesting, results are obtained regarding the Fourier spectrum of the solutions of this class of Riemann-Hilbert problems. The Wiener-Hopf factors of G are explicitly obtained, which allow us to establish invertibility criteria and formulas for the inverse of the associated operator.

Stationary solutions of selection mutation equations

Sílvia Cuadrado

Universitat Autònoma de Barcelona

A way of modelling biological evolution consists in considering densities of individuals with respect to evolutionary variables. This gives rise to selection mutation equations. We study the existence of stationary solutions of these equations and also their behavior when the mutation is small. The technics mainly involve positive semigroup theory and the infinite dimensional version in Banach lattices of the Perron Frobenius theorem.

Fredholm properties of a class of Toeplitz operators

Cristina Diogo

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Departamento de Métodos Quantitativos
Lisboa

In this talk the Riemann-Hilbert problem with matrix coefficient $G \in (L^\infty(\mathbb{R})^{2\times 2}$ is considered, assuming the existence of a non trivial solution (ϕ_+, ϕ_-) with $\phi_\pm$ belonging to the Hardy spaces $H_\infty(\mathbb{C}^\pm)$ and such that ϕ_+ or ϕ_- vanishes in some point of the corresponding half-plane $\mathbb{C}^+$ or $\mathbb{C}^-$, respectively. The results are used to study the Fredholm properties of Toeplitz operators with such a matrix symbol G in the Hardy spaces $H_2(\mathbb{C}^\pm)$.

Logaritmic convolution condition for homogeneous Calderón-Zygmund kernel

Petr Honzík

University of Missouri

Homogeneous Calderón-Zygmund kernel is a convolution kernel of the type

$$K_\Omega(x) = \frac{\Omega(x/|x|)}{|x|^n},$$

where Ω is an integrable function with mean zero on the unit sphere $\mathcal{S}^{n-1}$. It is well known that the corresponding convolution operator is bounded on L^2 if and only if the convolution

$$\int_{S^{n-1}} \Omega(\theta) \log \frac{1}{|\xi \cdot \theta|} d\theta \qquad (1)$$

is essentially bounded. We provide an example that shows that on L^p, $p \neq 2$, the condition (1) is no longer sufficient.

Sobolev embeddings

Jan Kalis

Florida Atlantic University

We prove a sharp Sobolev embedding with a zero boundary condition on a metric space which generalizes the real-case embedding. Using this new technique, we are able to prove a new sharp embedding on Lipschitz domains of $\mathbb{R}^n$ without the boundary condition. We also find the best constant for the real-case embedding and prove that this constant is not attained.

Compactness of integral operators on rearrangement-invariant spaces

Eva Kaspříková

Charles University, Prague

We study compactness of integral operators on Banach function spaces endowed with rearrangement–invariant norms. We develop a general method for the characterization of compactness of such operators.

Characterization of rearrangement invariant spaces with fixed points for the Hardy–Littlewood maximal operator

Joaquim Martín

Universitat Autònoma de Barcelona

We characterize the rearrangement invariant spaces for which there exists a non-constant fixed point, for the Hardy–Littlewood maximal operator. The main result that we prove is that the space $L^{\frac{n}{n-2},\infty}(\mathbb{R}^n) \cap L^\infty(\mathbb{R}^n)$ is minimal among those having this property.

Chaotic multipliers on spaces of operators

Alfred Peris[15]

Universitat Politècnica de València

We use tensor product techniques to study hypercyclicity and chaos of multipliers defined on certain operator ideals. An operator $T : X \to X$ on a Banach space X is hypercyclic if there are vectors $x \in X$ whose orbit $\{x, Tx, T^2x, \dots\}$ is dense in X. T is chaotic in the sense of Devaney if, moreover, the set of periodic vectors of T is dense in X. We also obtain the first examples of outer multipliers on a Banach algebra which are chaotic.

On norms of operators over spaces of abstract-valued functions

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We introduce a wide range of operators over classes of analytic functions with codomains in normed linear spaces, and study extremal problems for these operators. Several classical results for scalar-valued functions including the well-known Landau coefficient problem have been generalized for function spaces and operators under consideration.

[15]joint work with J. Bonet and F. Martínez Giménez

New results on restriction of multipliers

Salvador Rodríguez

Universitat de Barcelona

The theorem of K. de Leeuw on restriction of Fourier multipliers says that if $\mathbf{m}$ is a multiplier for $L^p(\mathbb{R})$, then $(\mathbf{m}(n))_{n\in\mathbb{Z}}$ is a Fourier multiplier for $L^p(\mathbb{T})$.

In this talk, we extend this result to general rearrangement invariant function spaces. The techniques used are the so called *Transference methods* due to R.R. Coifman and G. Weiss.

On Frechet norms and Mazur's Intersection Property

Jan Rychtář

Charles University, Prague; University of North Carolina at Greensboro

We show that a Banach space admits an equivalent norm with Mazur's intersection property if it contains a subspace having (1) a Frechet smooth norm and (2) a dual with the full weak* density. In the proof we will extensively use biorthogonal systems and results of M. Jimenez Sevilla and J. P. Moreno on spaces with Mazur's interesction property.

Regularity and asymptotic behaviour of the local time for the d-dimensional fractional Brownian motion with N-parameters

Josep Lluís Solé

Universitat Autònoma de Barcelona

We give the Wiener-Ito chaotic decomposition for the local time of the d-dimensional fractional Brownian motion with N-parameters. We study its smoothness in the Sovolev-Watanabe spaces and its asymptotic behaviour when the space variable tends to zero and the time variable tends to infinity.

Optimality of Sobolev Embeddings on R^d

Jan Vybíral

FSU Jena, Germany

We study the Sobolev embedding and its optimality on R^d. In general, we follow the approach of Edmunds, Kerman and Pick to the optimality of Sobolev embeddings on bounded domains. This time, new phenomena leading to the study of Hardy operators on nontrivial cones of positive functions appear.

Section Ring and Module Theory

Annihilator chain conditions in polynomial rings

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We study the behavior of annihilator chain conditions when passing from a ring R to the polynomial ring $R[x]$ and focus in the particular case of finite dimensional rings, that is, the Goldie condition. Hence, for any finite field K, we give an example of a commutative Goldie K-algebra R for which $R[X]$ is not Goldie. This generalizes a result of J.W. Kerr.

Cotorsion pairs and the Mittag-Leffler condition

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Let $\mathcal{C}$ be an abelian category with arbitrary direct sums. Let $(C_n)_{n\in\mathbb{N}}$ be a sequence of compact objects in $\mathcal{C}$, and let $(f_n\colon C_n \to C_{n+1})_{n\in\mathbb{N}}$ be a sequence of morphisms. Then we have the exact sequence

$$0 \to \oplus_{n\in\mathbb{N}} C_n \xrightarrow{\phi} \oplus_{n\in\mathbb{N}} C_n \to \varinjlim C_n = A \to 0$$

with $\phi\epsilon_n = \epsilon_n - \epsilon_{n+1} f_n$ for any $n \in \mathbb{N}$ and $\epsilon_n\colon C_n \to \oplus_{n\in\mathbb{N}} C_n$ denoting the canonical morphism.

Let $\mathcal{B}$ be a subcategory of $\mathcal{C}$ closed under direct sums. We show that the inverse system

$$(\mathrm{Hom}_{\mathcal{C}}(C_n, B); \mathrm{Hom}_{\mathcal{C}}(f_n, B))_{n\in\mathbb{N}}$$

[16]joint work with Ferran Cedó

[17]joint work with Silvana Bazzoni

is Mittag-Leffler for any object $B \in \mathcal{B}$ if and only if the morphism $\mathrm{Hom}_{\mathcal{C}}(\phi, B)$ is onto for any object $B \in \mathcal{B}$.

This result has some interesting consequences when applied to cotorsion pairs in module categories. For example, it is the key tool in showing that 1-dimensional tilting modules are of finite type. That is, if T_R is a tilting module over a ring R then there exists a set $\mathcal{S}$, consisting of finitely presented right R-modules of projective dimension at most one, such that

$$\mathrm{Ker}\,\mathrm{Ext}^1_R(T_R, -) = \bigcap_{S \in \mathcal{S}} \mathrm{Ker}\,\mathrm{Ext}^1_R(S, -).$$

Three uniform properties in noetherian rings

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In this talk we discuss the uniformity property for the Artin-Rees lemma, the relation type and the integral degree. We outline the way they are related to each other and present some uniform bounds. In doing so we will need to present part of the results in terms of the intersection of two overrings. Finally, we suggest some open problems related to the subject.

Remarks on the problem of Matlis

Pavel Příhoda

Charles University, Prague

Recall that the Matlis' problem is the question "Is the class of direct sums of indecomposable injective modules closed under direct summands?" We show a way how to prove the positive answer over hereditary rings.

[18] joint work with José Ma. Giral

Representations of distributive algebraic lattices by lattices of two-sided ideals of rings, resp. submodule lattices of modules

Pavel Růžička

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We will present some results concerning representations of algebraic lattices in ideal lattices of rings, resp. submodule lattices of modules. We prove that any algebraic lattice which can be represented as the lattice of two-sided ideals of some ring can be represented as the submodule lattice of some module as well. Then we focus on algebraic distributive lattices. An algebraic distributive lattice is determined by the semilattice of its compact elements and these semilattices correspond to distributive $\langle \vee, 0 \rangle$-semilattices. We present the following results:

- Every distributive semilattices which is closed under finite meets is isomorphic to the semilattice of finitely generated two-sided ideals of some locally matricial algebra.
- Every countable distributive semilattices is isomorphic to the semilattice of finitely generated two-sided ideals of some locally matricial algebra.
- Every distributive semilattices of size at most $\aleph_1$ is isomorphic to the semilattice of finitely generated two-sided ideals of some von Neumann regular ring but there is a distributive semilattices of size $\aleph_1$ which is not isomorphic to the semilattice of two-sided ideals of any unit-regular ring, in particular, of any locally matricial algebra.
- There is a distributive semilattices of size $\aleph_2$ which is not isomorphic to the semilattice of finitely generated submodules of any module.

Embedding domains in division rings

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Suppose we have a domain R embedded in a division ring E. We define inductively: $Q_0(R,E) = R$, and for $n \geq 0$,
$Q_{n+1}(R,E) = \begin{matrix}\text{subring of } E\\ \text{generated by}\end{matrix} \{r, s^{-1} \mid r, s \in Q_n(R,E),\ s \neq 0\}$.
Then $D = \bigcup_{n=0}^{\infty} Q_n(R,E)$ is the smallest division ring that contains R inside E. We define $h_E(R)$, the *inversion height* of R inside E, as ∞ if there is no $n \in \mathbb{N}$ such that $Q_n(R,E)$ is a division ring. Otherwise,

$$h_E(R) = \min\{n \mid Q_n(R,E) \text{ is a division ring}\}.$$

Let K be a commutative field. Suppose $\alpha\colon K \to K$ is a ring homomorphism which is not onto. Let $t \in K \setminus \alpha(K)$ and $k = \{a \in K \mid \alpha(a) = a\}$. Consider the skew polynomial ring $K[x;\alpha]$. It was proved by Jategaonkar that the k-algebra generated by x and $y = tx$ is a free k-algebra $k\langle x, y\rangle$. We call these embeddings *Jategaonkar embeddings.*

We prove:

(*i*) Jategaonkar embeddings have at most inversion height 2. And there are examples of Jategaonkar embeddings of height one and two.

(*ii*) If there is an embedding of the free algebra on two generators of height $1 \leq n \leq \infty$, then there exists an embedding of the free algebra on an infinite number of generators of inversion height n.

(*iii*) Let R be the free algebra or the free group algebra on $2 \leq m \leq \infty$ generators. We use examples in (*i*) to obtain embeddings of R of inversion height 1 or 2.

(*iv*) In a Jategaonkar embedding D is never the universal field of fractions of R.

[19]joint work with D. Herbera

A characterization of (co-)tilting cotorsion pairs and some independency results

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In the first part of my talk I shall present certain consequences of recent works of Šťovíček and Trlifaj; in particular an improvement of the existing characterization of tilting and cotilting cotorsion pairs. One of the sufficient conditions for a cotorsion pair to be (co-)tilting, completeness, is shown to be redundant.

Is it consistent with ZFC that the cotorsion pair, (A, B), generated by a set of modules is complete? This problem remains still unsolved. However, if we put some extra assumptions on the cotorsion pair (for example 'A is closed under pure submodules', but there are others), then it is possible to prove, assuming V = L, that (A, B) is cogenerated by a set, hence complete. This is (in brief) a content of the second part of my talk.

References

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[3] J. Šťovíček, All n-cotilting modules are pure-injective, to appear in Proc. Amer. Math. Soc.

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Closure properties of tilting and cotilting classes

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Let R be any (associative, unital) ring and $n \geq 0$ a natural number. Then any n-tilting as well as any n-cotilting class within the class of all (right R-) modules is *definable*, that is, closed under direct products, direct limits and pure submodules.

In the tilting case, the notions of a class of finite and countable type are used, [4]. A class $\mathcal{C}$ is of *countable* (*finite*) *type* if there is a set of countably (finitely) presented modules

[20]joint work with Silvana Bazzoni and Jan Trlifaj

$\mathcal{S}$ such that $\mathcal{C} = \mathcal{S}^{\perp_1} = \operatorname{Ker}\operatorname{Ext}^1_R(\mathcal{S}, -)$. The proof involves two steps. First, any n-tilting class $\mathcal{T}$ is proved to be of countable type using set-theoretic methods, [8]. Next, $\mathcal{T}$ is proved to be of finite type, generalizing results in [6], which already implies the definability of $\mathcal{T}$.

In the cotilting case, the problem was reduced to the following conjecture in the paper [2]:

If U is a module and ${}^{\perp_1}U = \operatorname{Ker}\operatorname{Ext}^1_R(-, U)$ is closed under direct products and pure submodules, then ${}^{\perp_1}U$ is closed under direct limits.

The proof of the latter assertion presented here is given in [7] by analyzing the cokernel of $M \hookrightarrow PE(M)$, where M is a module and $PE(M)$ is a pure-injective hull of M.

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[4] S. Bazzoni, P. Eklof and J. Trlifaj, *Tilting cotorsion pairs*, to appear in Bull. London Math.

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[8] J. Šťovíček and J. Trlifaj, *All tilting modules are of countable type*, preprint.

Infinite dimensional tilting modules and their applications

Jan Trlifaj

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We will introduce (infinite dimensional) tilting and cotilting modules, discuss their relation to the classical finite dimensional case, and present some of their recent applications to solutions of the finitistic dimension conjectures in particular cases [1], [3], and to the structure of Matlis localizations [2].

References

[1] L. Angeleri, D. Herbera, and J. Trlifaj, *Tilting modules and Gorenstein rings*, Forum Math. (to appear).

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