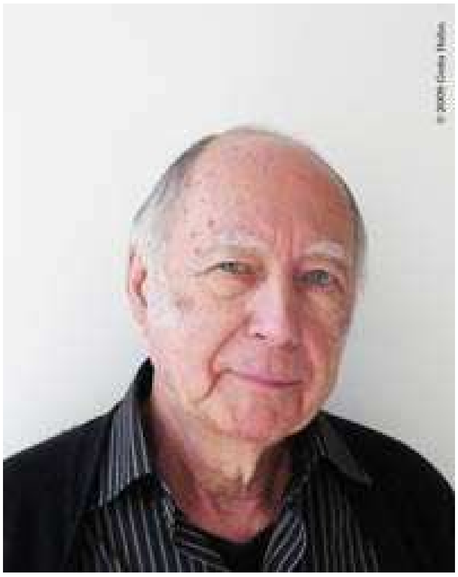
Algebraic Graph Theory

Gert Sabidussi is 80



Dubrovnik 2009

Introduction

This booklet is the collection of abstracts for the Algebraic Graph Theory meeting to be held in Dubrovnik, June 1-7, 2009. Apart from the rapidly developing field we have yet another motivation to organize this meeting: the eightieth birthday of Gert Sabidussi, who is one of the founders of Algebraic Graph Theory. Viewing this it is not very surprising that the response to our invitation was overwhelming and the quality of the meeting is reflected by this volume. We hope that this meeting will be a lasting tribute to his activity as a scientist, teacher and colleague.

This conference is supported by Steiermärkische Landesregierung, Montanuniversität Leoben, School of Computing Science, Simon Fraser University, Canada, EMS-CDC (Committee for Developing Countries of the European Mathematical Society), ITI-DIMATIA (Prague).

This volume was produced under technical assistance of David Hartman (ITI Charles University Prague).

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On Eigenvalues of the Vertex and Edge PI Matrices of a Graph.

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Let G be a graph. In this talk, at first the vertex and edge PI-matrices of G are introduced. Then we use the eigenvalues of these matrices to bound some graph invariants of G.

Transformations of edge colorings of graphs

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We discuss the following problem: Let $L_t(G)$ be the set of all proper edge colorings of a graph G with colors $1, 2, ..., t$. Find a system S of trans- formations such that for any $f, g \in L_t(G)$ there exists a sequence $f_1, \ldots, f_k$ of proper edge colorings in $L_t(G), k = k(f, g) \geq 2$, where $f_1 = f, f_k = g$ and f_{i+1} is obtained from f_i by one of the transformations in $S, i = 1, \ldots, k-1$.

Some recent results about vertex-transitive graphs.

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Some rather recent results about vertex-transitive graphs will be presented. This will include a presentation of highly arc transitive graphs and a discussion about the growth and separations in finite and infinite graphs.

A compactness result for Hamilton circles in locally finite graphs.

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Let G be a countably infinite graph. Following Diestel and Kühn, we define a topological space on G, denoted $|G|$, which corresponds to the Freudenthal compactification in the case that G is locally finite. A Hamilton circle in $|G|$ is a homeomorph of the unit circle. We introduce a finite sequences of graph G_n^* with the following two properties:

- $G_n^* \to G$ as $n \to \infty$;
- G is hamiltonian if and only if for some fixed m, G_n^* is hamiltonian for all $n \geq m$.

As a corollary we obtain that every 7-connected, claw-free, locally finite graph is hamiltonian.

This work comes from the Master's thesis of Funk.

The Abelian Sandpile Model

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In an unusual confluence of concepts emerging from a diverse set of areas, the Abelian Sandpile Model (ASM) offers a rich structure that has been studied for two decades by communities in statistical physics, probability theory, algebraic combinatorics, theoretical computer science, discrete dynamical systems.

The ASM, also referred to as a "chip firing game," associates a variety of remarkable structures with a simple diffusion process over graphs described by the reduced Laplacian.

After a general introduction to the ASM, we shall present recent combinatorial results and discuss open algorithmic questions.

Graph varieties

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There are several concepts of “variety” in graph theory. A natural one is to consider closure under two operations, graph multiplication and retraction, where one has a choice for the kind of product employed (strong, direct, or weak Cartesian). Absolute retracts come into play when strong or direct products of paths and their retracts are considered. With weak Cartesian products and retracts, the smallest nontrivial variety is that of median graphs, which would also be generated in terms of weak Cartesian multiplication and gated amalgamation. The latter concept is fruitful for generating graphs of certain polytopal complexes, for which the intervals $I(v, w)$ comprising all vertices lying on shortest paths between two vertices v and w essentially determine the structure of the complex (as a join space). This geometric avenue can be put into an entirely algebraic framework featuring a ternary operation $u, v, w \to (uvw)$ on the vertex set. In many cases the graphs in question are “apiculate”, that is, the intersection of two intervals $I(u, v)$ and $I(u, w)$ is an interval bounded by u and (uvw). Then equational classes with respect to this operation can be considered and their subdirectly irreducibles be characterized. For the class of weakly median graphs and their subvarieties this program works perfectly and also provides the geometric realizations as polytopal complexes in a number of cases.

Using product-like structures in sparse-matrix computations

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Computational grids for numerical simulation are frequently isomorphic—at least locally—to a graph product. Depending on the discretization stencil, this may be a cartesian, a strong, or a "triangulated" cartesian product. Numerical procedures such as matrix-vector multiplication or factorization profit from exploiting this structure. We present algorithms for recognizing product-type structures in computational grids and discuss feasibility, uniqueness and complexity. We show implementations of sparse-matrix multiplication kernels that use the structural information to save space and computational costs.

A generalization of Hungarian method and Hall's theorem with applications

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We consider various problems concerning covers and semi-matchings in bipartite graphs, which generalize the classical problem of determining (whether there is) a perfect matching in a bipartite graph. We present a vast generalization of Hall's marriage theorem, and an algorithm that solves the problem of determining a lexicographically minimum semi-matching of a bipartite graph. The study was motivated by problems in the design of optimal CDMA-based wireless sensor networks. It is a joint work with Drago Bokal and Janja Jerebic.

On $\mathcal{C}$-ultrahomogeneous graphs and digraphs

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The notion of a $\mathcal{C}$-ultrahomogeneous graph, due to Isaksen et al., is adapted for digraphs and studied for the twelve cubic distance transitive graphs, with $\mathcal{C}$ formed by the g-cycles and the $(k-1)$-paths, where g = girth and k = arc-transitivity. Excluding the Petersen, Heawood and Foster (90 vertices) graphs, one can go further by considering the $(k-1)$-powers of the g-cycles under orientation assignments provided by the initial study. This allows the construction of fastened $\mathcal{C}$-ultrahomogeneous graphs with $\mathcal{C}$ formed by copies of K_3, K_4, C_7 and $L(Q_3)$, for the Pappus, Desargues, Coxeter and Biggs-Smith graphs. In particular, the Biggs-Smith graph yields the non-line-graphical Menger graph of a self-dual (1024)-configuration, a K_3-fastened $\{K4, L(Q3)\}$-ultrahomogeneous graph; this contrasts with the self-dual (424)-configuration of [1], whose non-line-graphical Menger graph is K_2-fastened $\{K_4, K_{2,2,2}\}$-ultrahomogeneous. Among other results, a strongly connected C_4-ultrahomogeneous digraph on 168 vertices and 126 pairwise arc-disjoint 4-cycles is obtained, with regular indegree and outdegree 3 and no circuits of lengths 2 and 3, by altering a definition of the Coxeter graph via pencils of ordered lines of the Fano plane in which pencils are replaced by ordered pencils.

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Ends, earrings, and limits of free groups

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We characterize the fundamental group of a locally finite graph G with ends in two ways: combinatorially, as a group of infinite words, and algebraically, as a subgroup in the inverse limit of the free groups $\pi_1(G')$ with $G' \subset G$ finite. As an application, we can now describe the topological cycle space of G by a singular-type homology theory that works for any locally compact Hausdorff space X with a given compactification $\hat{X}$. This homology assigns a special role to the 'boundary points' in $\hat{X} \setminus X$, similar to the crucial role which the ends of a graph play in its topological cycle space.

Vertex PI Energy of Graphs

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Let G be a graph and $n_u(e)$ be the number of vertices closer to u than v and $n_v(e)$ be the number of vertices closer to v than u, where $e = uv \in E(G)$. Recently, the author introduced the notion of the vertex PI matrix of a graph. The aim of this talk is to find some bound for vertex PI energy of graphs.

Keywords and phrases: Vertex PI energy, Vertex PI Index.

On Barnette's conjecture

Tomás Feder and Carlos Subi

Barnette's conjecture is the statement that every 3-connected cubic planar bipartite graph is Hamiltonian. Goodey showed that the conjecture holds when all faces of the graph have either 4 or 6 sides. We generalize Goodey's result by showing that when the faces of such a graph are 3-colored, with adjacent faces having different colors, if two of the three color classes contain only faces with either 4 or 6 sides, then the conjecture holds. More generally, we consider 3-connected cubic planar graphs that are not necessarily bipartite, and show that if the faces of such a graph are 2-colored, with every vertex incident to one blue face and two red faces, and all red faces have either 4 or 6 sides, while the blue faces are arbitrary, provided that blue faces with either 3 or 5 sides are adjacent to a red face with 4 sides (but without any assumption on blue faces with $4, 6, 7, 8, 9, \ldots$ sides), then the graph is Hamiltonian. The approach is to consider the reduced graph obtained by contracting each blue face to a single vertex, so that the reduced graph has faces corresponding to the original red faces and with either 2 or 3 sides, and to show that such a reduced graph always contains a proper quasi spanning tree of faces. In general, for a reduced graph with arbitrary faces, we give a polynomial time algorithm based on spanning tree parity to decide if the reduced graph has a spanning tree of faces having 2 or 3 sides, while to decide if the reduced graph has a spanning tree of faces with 4 sides or of arbitrary faces is NP-complete for reduced graphs of even degree. As a corollary, we show that whether a reduced graph has a noncrossing Euler tour has a polynomial time algorithm if all vertices have degree 4 or 6, but is NP-complete if all vertices have degree 8. Finally, we show that if Barnette's conjecture is false, then the question of whether a graph in the class of the conjecture is Hamiltonian is NP-complete.

The Dirichlet problem in a network of finite total resistance

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It is proved that every locally finite electrical network of finite total resistance admits a unique current.

Coloring Vertices and Edges of a Path by Nonempty Subsets of a Set

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A graph G is strongly set colorable if $V(G) \cup E(G)$ can be assigned distinct nonempty subsets of a set of order n, where $|V(G)| + |E(G)| = 2^n - 1$, such that each edge is assigned the symmetric difference of its end vertices. The principal result is that the path $P_{2^{n-1}}$ is strongly set colorable for $n \geq 5$, disproving a conjecture of S.M. Hegde. We also prove another conjecture of Hegde on a related type of set coloring of complete bipartite graphs.
(joint work with P.N. Balister and R.H. Schelp)

The Sabidussi conjecture

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The Sabidussi conjecture says that to every eulerian tour in an eulerian graph there exists a compatible cycle decomposition, an edge disjoint collection of cycles without transition in common with the eulerian tour and using every edge. I shall for instance show that the conjecture is true for bipartite regular graphs, even though I now am fully convinced that it is false in general.

An aspect of direct product cancellation of digraphs

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We investigate expressions of form $A \times C \cong B \times C$ involving direct products of digraphs. Lovász gave exact conditions on C for which it necessarily follows that $A \cong B$. We are here concerned with a different aspect of cancellation. Given digraphs A and C, we classify digraphs A' for which $A \times C \cong A' \times C$. This leads to exact conditions on A such that any expression $A \times C \cong B \times C$ necessarily implies $A \cong B$.

On a Local Covering Algorithm for Approximate Products

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In several practical applications graphs arise as pertubed product structures. We will consider the *strong* product of graphs which has the pleasant property that neighborhoods in the product are products of neighborhoods of the factors. Thus the idea of a local covering algorithm is a natural approach towards the recognition of approximate products.

We introduce the notion of a *backbone* $\mathcal{B}(G)$ for a given graph G, defined as the set of vertices that have a strictly maximal closed neighborhood in G. Moreover, we will show that sufficient information for a local covering algorithm that decomposes graphs into their prime factors is already included in neighborhoods of vertices $v \in \mathcal{B}(G)$, for an extensive class of graphs.

Mosaics, even triangulations and quadrangulations of the sphere

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We define a mosaic to be a plane graph where every face is either a triangle or a quadrangle. We consider vertex coloring problems of mosaics and show relations to nowhere zero-flow problems. Thereby, we obtain a planar understanding of snarks and certain snark constructions. Moreover, we characterize those mosaics which can be extended to an even triangulation, i.e. a triangulation where every vertex has even degree, by adding a diagonal-edge into every quadrangle. This generalizes a theorem on quadrangulations, i.e. mosaics without triangles, by H. Zhang. Finally we improve a theorem of B. Mohar on quadrangulations. We show for every quadrangulation Q on the sphere and for every proper vertex 4-coloring of Q (using colors from $\{1, 2, 3, 4\}$) the following: the number of quadrangles whose vertices are colored in clockwise order $(1, 2, 3, 4)$ equal the number of quadrangles whose vertices are colored in anticlockwise order $(1, 2, 3, 4)$; the analog holds for every permutation of $(1, 2, 3, 4)$.

A remark concerning graph reconstruction

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The study of symmetry in graphs has a long tradition. It frequently is limited to using or investigating graph isomorphisms and in particular automorphisms. Early attempts of breaking with that tradition focus on hypomorphisms, isomorphisms into the complement, or graph homormorphisms. It appears that the least knowledge so far could be obtained regarding the reconstruction conjecture. I aim at showing in the talk that a graph is reconstructible if it has a vertex of degree one.

Graphs and transitive permutation groups

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We consider graphs arising from finite transitive groups. Let G be a finite transitive group on a finite set X. We call an orbit of G in its natural action on a set $X \times X$ by an *orbital* of G, and their number by rank of G.

Let Δ be an orbital, then $\Delta(x) = \{y \mid y \in X, \quad (x, y) \in \Delta\}$ and $\Delta(x)^g = \Delta(x^g)$ for all $x \in X$. Moreover $\Delta' = \{(y, x) \mid (x, y) \in \Delta\}$ is an orbital also. Orbital is symmetric if $\Delta = \Delta'$. A group G has a symmetric orbital different from diagonal if and only if $|G|$ is even.

So for any finite transitive permutation group of even order G acting on a finite set X we can define a graph $\Gamma = (X, \Delta)$, where Δ be a symmetric orbital.

Our aim to characterize such graphs by these properties.

Bipancyclic properties of Cayley graphs generated by transpositions

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We investigate the conditions of bipancyclic properties for Cayley graphs generated by transpositions. Bipancyclic property is pancyclic property for bipartite graphs. To obtain our result, we introduce a certain hamiltonian property , called 2-edge hamiltonian. We focus on Cayley graphs on the symmetric group $\mathfrak{S}_n$, since such Cayley graphs are wide graph classes, including star graphs and bubble sort graphs. Then our results includes the known results for these graphs.

The Fibonacci dimension of a graph

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The Fibonacci dimension $\mathrm{fdim}(G)$ of a graph G is introduced as the smallest integer f such that G admits an isometric embedding into the f-dimensional Fibonacci cube. We give bounds on the Fibonacci dimension of a graph in terms of the isometric dimension, denoted $\mathrm{idim}(G)$, and the lattice dimension; provide a combinatorial characterization of the Fibonacci dimension using properties of an associated graph; and establish the Fibonacci dimension for certain families of graphs. Algorithmic aspects are also considered. For instance, it is polynomial to decide whether $\mathrm{fdim}(G) = 2\,\mathrm{idim}(G) - 1$ but NP-complete whether $\mathrm{fdim}(G) = \mathrm{idim}(G)$. Approximation algorithms are also mentioned.

This is a joint work with Sergio Cabello (University of Ljubljana) and David Eppstein (University of California).

Monoids and graphs

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We start with one of Gert Sabidussis's ideas, who related the automorphism group of the lexicographic product of two graphs to the wreath product of the two automorphism groups. After the developing theory of wreath products for monoids similar questions can be asked for various endomorphism monoids of graphs. I present

1. some concepts of graph endomorphisms and give some problems and some answers to Sabidussi type questions
2. categorical interpretations of graph operations with the help of different homomorphisms.
3. algebraic properties of endomorphism monoids like regular, completely regular etc.
4. Cayley graphs of monoids
5. the genus of semigroups parallel to the genus of groups.

Biembeddings of Cayley tables of Abelian groups

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In a face 2-colourable triangular embedding of $K_{n,n,n}$, the colour classes correspond to Latin squares. We prove that, with the single exception of the 2-group $\mathcal{C}_2^2$, the Cayley table of each Abelian group appears as a colour class in a face 2-colourable triangular embedding of a complete regular tripartite graph in an orientable surface.

Topological representations of planar partial cubes

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Partial cubes (isometric subgraphs of hypercubes) form a well studied class of graphs with nice metric properties.They appear in diverse areas, like: computer science, mathematical chemistry, mathematical biology, social sciences, psychology. In mathematics they appear, besides in graph theory and combinatorics, also in topology, algebra and computational geometry.

Tope graphs of oriented matroids are examples of partial cubes. One of the fundamental results about oriented matroids is the topological representation theorem. Topological representation theorem for planar partial cubes will be presented. Different realizations of a given partial cube and their connection to the face lattice of a partial cube will be also discussed.

Hardness of approximation of the bipartite clique problem

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Consider the following problem usually called the bipartite clique problem: given a bipartite graph $G = (V, U, E)$, maximize k such that there exist $V_R \subseteq V$ and $U_R \subseteq U$, each of size k and the subgraph of G induced on the set of vertices $V_R \cup U_R$ is a complete bipartite graph. This problem is NP-hard and is one of the few problems for which we have neither a hardness of 'good' approximation result, nore a 'good' approximation algorithm. Some results on the approximability of the bipartite clique problem will be presented.

Semiregular Trees with Minimal Spectral Radius

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A semiregular tree is a tree where all non-pendant vertices have the same degree. Among all semiregular trees with a fixed order and degree, a graph with minimal spectral radius is a caterpillar. Counter examples show that the result cannot be generalized to the class of trees with a given (non-constant) degree sequence.

Injective colourings and homomorphisms of oriented graphs

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There are several possible definitions of an injective homomorphism of a digraph G to a digraph H. Each of them leads to a colouring parameter for oriented graphs by defining the injective oriented chromatic number of an oriented graph G to be the smallest number of vertices in an oriented graph H for which there is an injective homomorphism of G to H. One possible definition leads to the *good* (proper) and *semi-strong* (injective on in-neighbourhoods) colourings that first arose in the work of Courcelle, as well as a variation in which the colourings need not be proper. Another option leads to colourings (proper or not) that are injective on both in-neighbourhoods and out-neighbourhoods separately. We will consider these four possibilities and the associated colouring parameters that arise from them in terms of complexity, obstructions, critical graphs and bounds. This is based on joint work with Nancy Clarke, André Raspaud and Jacobus Swarts.

The median procedure on graphs

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A consensus function is a model to describe a rational way to obtain consensus among a group of agents or clients. The input of the function consists of certain information about the agents, and the output concerns the issue, about which consensus should be reached. The rationality of the process is guaranteed by the fact that the consensus function satisfies certain "rational" rules or "consensus axioms". A typical question in consensus theory is: which set of axioms characterizes a given consensus function. Our main focus is the median function on graphs: the input is the location of the clients in the graph, the output is the set of medians, i.e. the vertices that minimize the average distance to the locations of the clients. We survey old and new results. Median graphs play a major role.

Binary codes of the complements of the Triangular graphs.

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For an positive integer $n \geq 2$, the complement of the triangular graph $\overline{T(n)}$ has vertices the set $\Omega_n^2 := \{\{i, j\} : i, j \in \{1, 2, \ldots, n\}\}$ and adjcency defined by two vertices being adjacent if they have nothing in common. We examine the binary codes obtained from the adjacency matrices of these graphs, as a parallel to the binary codes of the triangular graphs $T(n)$ discussed in Heamers [4] and more recently in Key [6]. We find the automorphism group of the codess and also obtain their PD-sets.

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A census of edge-transitive planar tilings

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Recently Graves, Pisanski and Watkins have determined the growth rates of Bilinski diagrams of one-ended, 3-connected, edge-transitive planar maps. The computation depends solely on the edge-symbol $\langle p, q; k, l \rangle$ that was introduced by B. Grünbaum and G. C. Shephard in their classification of such planar tilings. We present a census of such tilings in which we describe some of their properties, such as whether the edge-transitive planar tiling is vertex- or face-transitive, self-dual, bipartite or Eulerian. In particular, we order such tilings according to the growth rate and count the number of tilings in each subclass.

Designs, Graph Decompositions and Quasigroups.

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The relationship between designs and graph decompositions on one hand, and quasigroups on the other, was explored by many researchers, e.g. Kotzig, Sabidussi, Lindner, Bryant and many others. We discuss various characterizations, both old and new, for several varieties of "graphical" quasigroups.

Automorphism groups and partitions of relational structures

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Due to results of Kechris, Pestov and Todorcevic various connections between actions of topological groups on compacta and relational Ramsey theory have become known.

On the other hand the actions of those groups on countable sets determine relational structures and to a certain extent their partition properties. A very general introduction into relational structures, homogeneous structures, their groups of automorphisms and the partitions of those structures determined by the automorphism groups will be given. Including a short outline of the above mentioned results of Kechris, Pestov and Todorcevic.

A Removal Lemma for Groups

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I will report on recent extensions of a result by Ben Green which states that if an equation has not many solutions in a set of a group then these solutions can be removed by eliminating few elements in the set. The proofs are based on the Removal Lemma for graphs and for hypergraphs. This is joint work with Dan Kral and Lluis Vena.

Large Cayley graphs and vertex-transitive non-Cayley graphs of given degree and diameter

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For any $d \geq 5$ and $k \geq 3$ we construct a family of Cayley graphs of degree d, diameter k, and order at least $k((d-3)/3)^k$. By comparison with other available results in this area we show that, for all sufficiently large d and k, our family gives the current largest known Cayley graphs of degree d and diameter k.

Nowhere-zero flows in Cayley graphs.

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It has been conjectured (Alspach et al., 1996) that every (2-connected) Cayley graph has a nowhere-zero 4-flow. Furthermore, if Tutte's 3-flow conjecture is true, then every Cayley graph of valency greater than 3 has a nowhere-zero 3-flow. We survey the known results about nowhere-zero flows in Cayley graphs and show that (i) the smallest potential counterexample to the former conjecture must be a cubic Cayley graph on a finite non-abelian simple or a "nearly" simple group, and (ii) that every Cayley graph of valency greater than 3 on a nilpotent group has a nowhere-zero 3-flow.

Exponential quadripartition: snarks and cubic non-snarks versus hypohamiltonicity

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It is shown that among n-vertex cubic nonhamiltonian and nonplanar graphs with girth 5 and cyclic edge connectivity 4, the intersection of two bipartitions: snarks – non-snarks and hypohamiltonian – non-hypohamiltonian, is a quadripartition into exponentially numerous classes provided that n is even and large enough. Thus the properties snark and hypohamiltonicity are proved to be essentially unrelated. Moreover, the constructed graphs are all strongly nonplanar, that is, all contain a subdivision of the Petersen graph.

Similarity of large graphs, convergence of graph sequences.

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In the lecture I give a short survey on some joint work with Crhristian Borgs, Jennifer Chayes, Laszlo Lovasz and Kati Vesztergombi and also on some results of Lovasz and Balazs Szegedy.

Many areas in mathematics , computer science, biology, physics etc. study properties of very large (deterministic or random) graphs or increasing graph sequences . Here the properties we are interested in are very often different from the questions we investigate for graphs with any fixed number of vertices .

The questions that first arise are:

- when are two large graphs similar
- when are two large graphs close to each other
- which and how local and global properties are related to each other
- how large graphs can be approximated by "small" , "simple" graphs.

For different classes of graphs (like e.g. for dense graphs, for sparse or "very" sparse graphs, for hypergraphs) the nature and difficulty of the problems are very different.

Here we consider dense graphs,we outline a part of a wide ongoing project, some of the results and problems (of a theory) which developed in the last few years in the center with Laszlo Lovasz .

We define several natural notions of convergence of graph sequences. We show that these notions of convergence are equivalent and a sequence is convergent if and only if it is Cauchy in the given (properly defined) distance of graphs. These provide to study the similarity large graphs, the closeness of both local and global properies.

The results are related e.g.to parameter and property testing, to Szemeredi's regular partition,spectra of graphs , closer and finer relations between local and global propeties; extremal (graph)problems.

Our results can also be viewed as a generalisation of some results for quasi random graphs.

Graphs, Grammars, and RNA Structures

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RNA molecules and their structures have become a central research topic in Molecular Biology and Bioinformatics during the last decade. From a mathematicsl point of view, "RNA secondary structures" are matchings M in a graph G_x of whose vertices are the individual nucleotides (letters) of the RNA sequence x. Edges connect vertices/nucleotides that satisfy biophysical pairing rules on the underlying nucleotide alphabet $\{A, C, G, T\}$. Legal structures satisfy certain additonal constraints. In the simplest case they form circular matchings, i.e., edges in M do not cross w.r.t. the given order of vertices along the RNA sequence. Each allowed matching is furthermore associated with an energy that can be computed additively from empirical local rules. The RNA folding problem thus consists of finding the matching with minimal energy.

Depending on the constraints on the matching, different types of folding problems arise, for which in many cases recursive solutions exist that can be expressed conveniently in terms of corresponding grammars, which in turn imply polynomial-time solutions of the optimization problems. The presentation will focus on variants of the RNA-RNA interaction problem. Although here we consider two RNA sequences, we again obtain matching problems of the same flavor.

Beautiful minds - and the game of Hex

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The game of Hex was discovered in 1942 by the Danish poet and designer Piet Hein, and independently a few years later by John Nash. The game has an interesting history, and it is also interesting from a mathematical point of view because it has very simple rules, but is very difficult to play well. The lecture presents the history of Hex in a broad context, and it gives some new Hex results and proofs.

Vertex-transitive graphs: between group theory and graph theory

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In the talk the situation when a connected locally finite graph admits a vertex-transitive group of automorphisms is considered. The situation naturally arises in many different contexts and is very important both for group theory and graph theory. In the first part of the talk we discuss a theory of tracks of graphs which is an effective instrument to study vertex stabilizers of graphs in vertex-transitive groups of automorphisms. (Note that, by a well-known result of Gert Sabidussi, for such a group the order of the stabilizer of a vertex is a natural measure of how far is this group action from the natural action of the group on its Cayley graph.) In the second part of the talk, for directed vertex-transitive graphs, the following question

is considered: What is the length of a shortest directed path starting in a fixed vertex of a given finite subset of the vertex set and ending outside of the subset? Applications of obtained results in combinatorial group theory are given.

Strongly regular $(96, 20, 4, 4)$ and $(96, 19, 2, 4)$ graphs from difference sets

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Strongly regular graphs with parameters $(96, 20, 4, 4)$ and $(96, 19, 2, 4)$ are constructed as Cayley graphs over groups of order 96. Our source is the list of 2607 so far constructed $(96, 20, 4)$ difference sets. For the computation we use GAP, the well-known system for computational group theory and in particular its part GRAPE for graph exploring.

On the Equation “Statistical Physics Equals Probabilistic Combinatorics Minus Graph Theory”

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In the last two decades combinatorialists and statistical physicists have discovered that they have much in common, in particular in the study of random configurations (such as colorings or independent sets) on graphs. But the physicsts seem to be concerned with particular graphs, which they call lattices, thus cutting out some of the interesting graph theory. It turns out that graph theory nonetheless comes back in in some quite surprising ways. We will how graphs reappear in two kinds of “hard constraint” systems and help shed light both on statistical physics models and on combinatorics itself.

Combinatorics of infinite words and graphs related to the fundamental group of fractals

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Historically, algebraic topology has been developed mainly with view on topological spaces which are manifolds. Nevertheless also for spaces like fractals the fundamental group contains much information. Such groups typically are quite complicated and little algebraic structure theory is known. In a cooperation with Shigeki Akiyama, Gerhard Dorfer and Jörg Thuswaldner we have developed a description of such fundamental groups involving projective limits of semigroups of words over increasing alphabets and infinite graph like structures. A typical example where our method applies is the Sierpinski gasket. (The research has been supported by the Austrian Science Foundation FWF, project S9612-N23.)

Context-free pairs of groups

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We take up some aspects of the important work of Muller and Schupp from 25 years ago. Let G be a finitely generated group and K a subgroup. Given a finite set of generators of G, the language $L(G, K)$ consists of all words over those generators that reduce to an element of K. The pair (G, K) is called context-free, if $L(G, K)$ is a context-free language. Context-freeness of (G, K) is independent of the specific choice of the generating set of G. We show that $L(G, K)$ is context-free if and only if the Schreier graph of (G, K) is a context-free graph in the sense of Muller and Schupp. Further basic properties of context-free pairs are given.

Some of the mathematics I have done in Montreal

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I have been a frequent visitor to my friends in Montreal, Canada, starting in the seventies of the previous century. I wish to describe a few of the results which I have obtained during these visits; they include:

1. some insights into the theory of neighborly families of convex polytopes;

2. some rational analogues to the Beckman-Quarles theorem about unit-distance mappings, and rational embeddings of sets;

3. a simple proof of the two-dimensional case of a lemma, due to M. Pouset, which is equivalent to the Brouwer Fixed-Point Theorem in the plane, and more.

Fault diameters of graph products and bundles

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Fault tolerance and transmission delay of networks are important concepts in network design. The notions are strongly related to connectivity and diameter of a graph, and have been studied by many authors. Wide diameter of a graph combines studying connectivity with the diameter of a graph. Diameter with width k of a graph G is defined as the minimum integer d for which there exist at least k internally disjoint paths of length at most d between any two distinct vertices in G. In the context of computer networks, wide diameters of Cartesian graph products have been recently studied [5, 6]. Cartesian graph bundles [7] is a class of graphs that is a generalization of the Cartesian graph products. We show that if G is a k_G-connected graph and $D_c(G)$ denotes the c-diameter of G, then $D_{a+b}(G) \leq D_a(F) + D_b(B)$, where G is a graph bundle with fiber $F = K_2$ over base $B = K_2, 0 < a \leq k_F$, and $0 < b \leq k_B$ [4]. Not surprisingly, there are analogous inequalities known for some related invariants including vertex- and edge-fault diameters [3, 1] and hence it is interesting to study the relationships among these obviously related notions. I will briefly report on some recent work with Iztok Banič and Rija Erveš [1, 2, 3, 4].

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Cycle covers – minimal contra pairs, Petersen chain and Hamilton weights

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Let G be a bridgeless cubic graph associated with an eulerian weight $w : E(G) \mapsto \{1, 2\}$. A family $\mathcal{F}$ of circuits is a *faithful circuit cover* of the eulerian weighted graph (G, w) if every edge e of G is contained in precisely $w(e)$ members of $\mathcal{F}$. A circuit C of G is *removable* if the graph obtained from G by deleting all weight 1 edges contained in C remains bridgeless.

An eulerian weighted graph (G, w) is a *contra pair* if it has no faithful circuit cover, and a contra pair is *minimal* if (G, w) has no removable circuit and, for every weight 2 edge e, the eulerian weighted graph $(G - e, w)$ has a faithful circuit cover. A contra pair (G, w) is e_0-*minimal* if, for a *given* weight 2 edge e_0, the eulerian weighted graph $(G - e_0, w)$ has a faithful circuit cover, and very removable circuit must contain the given edge e_0.

It was proved by Alspach and Zhang (1993) that if (G, w) is a minimal contra pair, then the graph G must *contain the Petersen graph as a minor.* It is further conjectured by Fleischner and Jackson (1988) that this graph G must be *the Petersen graph* itself.

In this project, it is proved that *Fleischner-Jackson conjecture is true if Hamilton weight conjecture is true.* This result is further extend that *if a contra pair (G, w) is e_0-minimal, then G must be a Petersen chain joining the endvertices of e_0 if Hamilton weight conjecture is true.* Those results are necessary steps in some approaches to the Cycle Double Cover conjecture (Szekeres 1973 and Seymour 1979).

Note that an eulerian weighted graph (G, w) is a Hamilton weight pair if every faithful circuit cover of (G, w) is a pair of Hamilton circuits. The Hamilton weight conjecture states that every 3-connected Hamilton weight pair is constructed from K_4 via a series of $Y - \Delta$-operations.

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