Preface

Welcome to the Czech-Slovak Conference on Combinatorics and Graph Theory 'Graphs 2003'.

The history of the Czech and Slovak graph-theoretical conferences dates back to 1961, when the first meeting, containing a section on Combinatorics and Graph Theory, took place at Liblice. Since 1969, the Czechoslovak (from 1993 Czech-Slovak) conferences on Combinatorics and Graph Theory have been held annually. Once in eight years, this conference becomes an international symposium, co-organized by all main Czech and Slovak Graph Theory centers.

The 2003 conference is organized by the Pilsen branch of the Institute of Theoretical Computer Science, Charles University (ITI), and by the Department of Mathematics, University of West Bohemia in Pilsen. The conference is held at Javorná, a village in the Šumava mountains, in the Southern part of the Czech Republic, on June 2-6, 2003. Support by the grant No. LN00A056 of the Czech Ministry of Education is gratefully acknowledged.

The scientific program of the conference consists of 50 min. lectures of invited speakers and of 25 min. contributed talks presented by other participants.

Invited speakers:
Stanislav Jendrol', Košice, Slovakia: Local properties of graphs embedded into surfaces
Mirka Miller, Newcastle, Australia: On summable graph labellings
Ladislav Nebeský, Praha: On the interval function of a connected graph
Moshe Rosenfeld, Tacoma, U.S.A.: Variations on Hamiltonian themes
Pavel Valtr, Praha: Extremal questions for geometric graphs
Heinz-Jürgen Voss, Dresden, Germany: On k-trestles in polyhedral graphs

This brochure contains abstracts of invited lectures and contributed talks as sent to us by the authors. Conference languages are Czech, Slovak and English.

We wish all participants a pleasant and fruitful stay at Javorná.

Pilsen, May 26, 2003

The organizers

Organizing committee:

Jan Brousek	Roman Kužel
Roman Čada	Zdeněk Ryjáček (chair)
Přemysl Holub	Libuše Tesková
Tomáš Kaiser	Milena Tuchanová

Contents

Note: If there are several coauthors, the speaker is always listed first in this table of contents.

History

A list of all previous Czechoslovak (from 1993 Czech-Slovak) conferences on Combinatorics and Graph theory.

1961 Liblice
1963 Smolenice (international)
1966 Smolenice
1969 Smolenice
1970 Modra
1971 Zlatá Idka
1972 Štiřín
1973 Staré Splavy
1974 Praha (international)
1975 Brno
1976 Smolenice
1977 Jenišov
1978 Zemplínská Šírava
1979 Nová Ves u Branžeže
1980 Pardubice
1981 Jablonec nad Nisou
1982 Praha (international)
1983 Zemplínská Šírava
1984 Kočovce
1985 Luhačovice
1986 Račkova dolina
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1988 Lazy pod Makytou - Čertov
1989 Hrubá Skála
1990 Prachatice (international)
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1993 Janov nad Nisou
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1996 SoláňČarták
1997 Chudenice
1998 Praha (international)
1999 Kočovce
2000 Liptovský Trnovec
2001 Sedmihorky
2002 Rejvíz

Labelings of graphs

Martin Bača

The modern topic of *labelings of graphs* is a relatively young, but very fruitful, part of graph theory: in the last 30 years there have appeared many papers on this topic.

We deal with *edge-antimagic labelings* for the special classes of graphs and *antimagic labelings* of type $(1, 1, 1)$ for the plane graphs.

On-line ranking of dodecahedron

Erik Bruoth

A k-ranking of a graph G is a mapping $\varphi : V(G) \to \{1, ..., k\}$ such that any path in G with endvertices x, y fulfilling $\varphi(x) = \varphi(y)$ contains an internal vertex z with $\varphi(z) > \varphi(x)$. The ranking number $\chi_r(G)$ of a graph G is the minimum k such that G has a k-ranking. On-line ranking number $\chi_r^*(G)$ of a graph G is the minimum k such that G has a k-ranking constructed step by step if vertices of G are coming and coloured one by one in an arbitrary order; when colouring a vertex, only edges between already present vertices are known. Here we present upper and lower bound of on-line ranking number for dodecahedron.

Paths and cycles with prescribed edges in hypercubes and dense sets

Rostislav Caha and *Václav Koubek*

This talk is devoted to spanning path and cycles with prescribed edges in dense sets. This topic was initiated by paper [1] due to the authors and extended in [2].

For a dense set $\mathcal{D}$, a given set of edges $\mathcal{F}$ and vertices A, B from $\mathcal{D}$ we show necessary conditions for finding a $\mathcal{D}$-spanning path $\mathbf{P}$ from A to B such that $\mathcal{F} \subseteq E(\mathbf{P})$. Similar conditions exist for a problem of finding $\mathcal{D}$-spanning cycle $\mathbf{C}$ in $\mathcal{D}$ such that $\mathbf{C}$ also contains all prescribed edges from $\mathcal{F}$.

References

[1] R. Caha and V. Koubek, *Hamiltonian cycles and paths with a prescribed set of edges in hypercubes and dense sets*, submitted to *J. Graph Theory*

[2] T. Dvořák, *Hamiltonian cycles with prescribed edges in the hypercube*, to appear

Short disjoint paths in locally connected graphs

Chuanping Chen, Roman Čada,
Tomáš Kaiser and Zdeněk Ryjáček

A well-known observation due to Chartrand and Pippert says that a connected, locally k-connected graph is $k+1$-connected. We discuss several related problems. Our aim is to find (under an assumption on local k-connectivity) a set of $k+1$ vertex or edge-disjoint paths of bounded length, particularly we are interested in paths of almost equal length. As a new graph parameter we introduce the concept of a k-diameter and its local variant.

Generalized H-coloring and H-covering of trees

Jiří Fiala, Pinar Heggernes,
Petter Kristiansen and Jan Arne Telle

We study $H(p,q)$-colorings of graphs, for H a fixed simple graph and p,q natural numbers, a generalization of various other vertex partitioning concepts such as H-covering. An H-cover of a graph G is a local isomorphism between G and H, and the complexity of deciding if an input graph G has an H-cover is still open for many graphs H. We show that the complexity of $H(2p,q)$-COLORING is directly related to these open graph covering problems, and answer some of them by resolving the complexity of $H(p,q)$-COLORING for all acyclic graphs H and all values of p and q.

Vertex magic total labelings of products of graphs

Dalibor Fronček, Petr Kovář and Tereza Kovářová

Vertex magic total labeling of a graph $G(V, E)$ is a bijection λ from the set $V \cup E$ to $\{1, 2, \ldots, |V| + |E|\}$ with the property that for each vertex the sum of the label of the vertex and the labels of all edges incident to that vertex is equal to a given constant k. That is, $\lambda(x) + \sum_{\{y|xy \in E\}} \lambda(xy) = k$ for every x in V.

We will present vertex magic total labelings for some new families of graphs, namely products of odd cycles and maybe some more, depending on our luck. If time permits, we may also mention some interesting connections between certain vertex labelings and tournament scheduling.

Neigbours distinguishing index of plane bipartite graphs

Keith Edwards, Mirko Horňák and Mariusz Woźniak

Let G be a finite simple graph with no component K_2. Let C be a finite set of colours and let $\varphi : E(G) \to C$ be a proper edge colouring of G. The *colour set* of a vertex $v \in V(G)$ with respect to φ, in symbols $S_\varphi(v)$, is the set of colours of edges incident with v. The colouring φ is *neighbours distinguishing* if $S_\varphi(u) \neq S_\varphi(v)$ whenever $u, v \in V(G)$ and $uv \in E(G)$. The *neighbours distinguishing index* of the graph G, in symbols $\mathrm{ndi}(G)$, is the smallest number of colours in a neighbours distinguishing colouring of G.

Evidently, $\mathrm{ndi}(G) \geq \Delta(G)$. It is easy to see that $\mathrm{n}di(C_5) = 5 = \Delta(C_5) + 3$. The neighbours distinguishing index has been introduced by Zhongfu, Liu and Wang. Among other things these authors conjecture that $\mathrm{ndi}(G) \leq \Delta(G) + 2$ for any graph $G \neq C_5$ with $|V(G)| \geq 3$. The conjecture has been confirmed by Balister, Győri, Lehel and Schelp for bipartite graphs.

Theorem. If G is a plane bipartite graph with $\Delta(G) \geq 12$, then $\mathrm{ndi}(G) \leq \Delta(G) + 1$.

Eccentric sequences and cycles in graphs

Alfonz Haviar, Pavel Hrnčiar and Gabriela Monoszová

The *eccentric sequence* of a connected graph G is a list of the eccentricities of its vertices in nondecreasing order. A sequence of positive integers is called eccentric if there is a graph having the considered sequence as the sequence of its eccentricities. An eccentric sequence is called *minimal sequence* if it has no proper eccentric subsequence with the same number of distinct eccentricities. A graph is said to be a *minimal graph* if its eccentric sequence is minimal. Minimal graphs with only one cycle are described. Moreover, all minimal eccentric sequences of the type $(r^{\alpha}, (r+1)^{\beta})$, $r \geq 6$, $\alpha+\beta \leq \frac{8r+5}{3}$ are found. These results are based on the theorem saying that if a graph G satisfies the following conditions:

(i) $\operatorname{rad} G = r$,

(ii) $\operatorname{diam} G \leq 2r-2$ and

(iii) $|V(G)| \leq 3r-2$,

then G contains a cycle of length 2r or 2r+1.

The minimal polynomial of $2\cos(\pi/p)$ and regular maps of large planar width

Mária Ipolyiová

An important problem in the theory of maps on orientable surfaces is to determine the smallest number of darts $\delta(m,n,r)$ of a regular map of type $\{m,n\}$ (i.e.,of face length m and vertex valence n) with planar width larger than r. An upper bound on $\delta(m,n,r)$ was derived in [1] using 3-dimensional linear representations of triangle groups $T(2,m,n) = \langle x,y;\ x^n = y^2 = (xy)^m = 1\rangle$. In this contribution we improve the bound of [1] for regular maps of type $\{p,3\}$ where $p \geq 11$ is prime. The method relies upon a detailed study of minimal polynomials of $2\cos(\pi/p)$.

References

[1] J. Širáň, Triangle group representations and costructions of regular maps, *Proc. London Math. Soc.* (3) 82 (2001) 513-532.

Local properties of graphs embedded into surfaces

Stanislav Jendrol'

In the talk we present a survey of recent results and open problems concerning subgraphs with restricted degrees of their vertices in graphs and multigraphs embedded in surfaces. Particularly we will speak about Kotzig type theorems for surfaces, light paths in different types of maps on surfaces and about light families of graphs in embedded graph.

Note: The talk in based on a recent joint research with M. Tuhársky (Košice) and H.J. Voss (Dresden).

2-walks in graphs

Tomáš Kaiser

A *2-walk* in a graph G is a spanning closed walk visiting each vertex of G at most twice. We survey results and conjectures on the existence of 2-walks, especially in connection to the toughness of G. Parts of the talk come from joint work with M. Kriesell and from discussions with J. Teska.

Power series in combinatorial enumeration

Martin Klazar

We will discuss a hierarchy of four classes of power series used in combinatorial enumeration: rational, algebraic, D-finite, and differentially algebraic power series. Several examples from combinatorics, theory of formal languages, number theory, and probability theory will be given.

Face 2-colourable triangulations of $K_{n,n,n}$ and biembeddings of Latin squares

Martin Knor

Face 2-colourable triangulations of $K_{n,n,n}$ correspond to biembeddings of Latin squares. This observation yields a simple proof of an existence of a regular face 2-colourable triangulation of $K_{n,n,n}$ in an orientable surface for any $n \geq 2$. Using a computer we found all biembeddings of Latin squares of order $n \leq 7$.

Equivalent versions of the 3-flow conjecture

Martin Kochol

The 3-flow conjecture of Tutte is that every graph without 1- and 3- edge cuts has a nowhere-zero 3-flow. We prove that this is equivalent with seemingly stronger and weeker statements. In particular, we show that this conjecture suffices to verify for 5-edge-connected graphs.

Decomposition of even graphs into closed trails

Mirko Horňák and Zuzana Kocková

For a graph G, let Lct(G) denote the set of all integers $l \leq 3$ such that there exists a closed trail of length l in G and let Sct(G) denote the set of all sequences $(l_1, l_2, \ldots, l_p)$ such that $l_i \in$ Lct(G), $i = 1, 2, \ldots, p$ and $\sum_{i=1}^{p} l_i = |E(G)|$. A graph is said to be even if the degrees of all its vertices are even. A connected even graph G is said to be *arbitrarily decomposable into closed trails* (ADCT for short) if, given any sequence $(l_1, l_2, \ldots, l_p) \in$ Sct(G), G can be edge-disjointly decomposed into closed trails $T_1, T_2, \ldots T_p$ of lengths $l_1, l_2, \ldots, l_p$, respectively.

Theorem 1 *If the graph $K_{p,q,r}$ with $p \leq q \leq r$ is* ADCT, *then $(p, q, r) = (1, 1, 3)$ or $p = q = r$.*

Proposition 2 *Graph* $K_{1,1,3}$ *is* ADCT.

Theorem 3 *If* $n \in \{1, 2, 3, 4\}$ *or* $n = 5 \cdot 2^i$*, where i is a nonnegative integer, then* $K_{n,n,n}$ *is* ADCT.

A note on the greedy algorithm for the unsplittable flow problem

Petr Kolman

In the Unsplittable Flow Problem (UFP), we are given a graph $G(V, E)$ with edge capacities prescribed by $c : E \to R_+$, and a set $T = \{(s_i, t_i) : 1 \leq i \leq k\}$ of requests with an associated demand d_i for each request. A feasible solution for the UFP is a subset of requests from T such that all of them can be routed in G without violating the capacity constraints, each request along a single path. The objective is to find a feasible solution with maximal total demand. In the Edge Disjoint Paths Problem (EDP), all edge capacities and all demands are one.

In a recent paper Chekuri and Khanna (SODA 2003) improved the analysis of the Greedy algorithm for the EDP and proved the same bounds also for the related Uniform Capacity UFP (that is, all edge capacities are one). Here we show that their ideas can be extended to get the same approximation ratio even for the more general UFP with *arbitrary* edge capacities. Specifically, we propose an $O(n^{2/3})$-approximation algorithm for undirected graphs and an $O(n^{4/5})$-approximation algorithm for directed graphs.

Coloring powers of chordal graphs

Daniel Král'

We prove that the second power of a chordal graph G can be colored by $O(\Delta^{3/2})$ colors where Δ is the maximum degree of G. This improves a previously known upper bound of order $\Delta^2/4 + O(\Delta)$. A construction of chordal graphs G such that $\chi(G) \geq \Omega(\Delta^{3/2})$ is also presented.

In general, we show that the k-th power G^k of a chordal graph G with maximum degree Δ is $O(\sqrt{k}\Delta^{(k+1)/2})$-degenerated for even values of k and $O(\Delta^{(k+1)/2})$-degenerated for odd ones. The bound proven for odd values of k is the best possible.

Geometric intersection graphs

Jan Kratochvíl

Intersection graphs of geometric objects, especially in the plane, have been intensively studied both for their practical motivation and interesting structural and algorithmic properties. We will survey recents results on string graphs, related abstract topological graphs and polygon circle graphs.

α-like labeling and drafting of the graphs

Michael Kubesa

We examine how we can construct from the trees with graceful or α-labeling non-symmetric trees with flexible q-labeling or α-like labeling which allow factorization of K_{2n} into spanning trees.

Every 4-connected line graph is hamiltonian if and only if it is hamiltonian connected

Roman Kužel, Liming Xiong

In 1986, C. Thomassen proposed the well-known conjecture that every 4-connected line graph is hamiltonian. We show that this conjecture is equivalent to the statement that every 4-connected line graph is hamiltonian connected.

On edge connectivity of cages

Yuqing Lin and Mirka Miller

A $(k;g)$-cage is a k-regular graph with girth g and with the least possible number of vertices. The $(k;g)$-cages have been studied extensively. However, the results have been far from satisfactory. For example, the problem of determining the value of the order of $(k;g)$-cages is unknown for most pairs of values, and it is extremely hard to find in the general case.

In this talk, I will present some new results regarding to the connectivity of $(k;g)$-cages.

In 1997, H. L. Fu *et al.* proved that all cages are 2-connected. Later both Daven together with Rodger, and Jiang together with Mubayi independently went on to provide different proofs that all cages are 3-connected. Recently Marcote et al. proved that all cages with $g \geq 9$ are 4-connected, cages with girth $g = 6, 8$ are k-connected, and most of cages with $g = 5$ are also k-connected. It follows that almost all cages are 4-edge-connected. Furthermore, H.L.Fu *et al.* conjectured that all simple $(k;g)$-cages are k-connected which also implies that all simple $(k;g)$-cages are k-edge-connected.

Regarding the edge-connectivity of $(k;g)$-cages, Wang *et al.* proved that all $(k;g)$-cages are k-edge-connected for g odd. Furthermore, Marcote and Balbuena have proved that all (k,g)-cages with odd girth are edge-superconnected, which means that all the minimal edge-cut sets are trivial.

We have showed recently that cages with even girth are k-edge-connected and also edge-superconnected. Combining this with previous results allows us to conclude that all $(k;g)$-cages are k-edge-connected, and furthermore, edge-superconnected.

Cayley maps and their exponents

L'ubica Líšková

A Cayley map is a Cayley graph embedded in an orientable surface in such a way that the cyclic order of generators is the same at each vertex. Properties of Cayley maps also depend on the distribution of inverses, that is, on the location of mutually inverse generators in the cyclic order.

Cayley maps are automatically vertex-transitive. Higher degree of symmetry can be induced by regularity and by the existence of exponents. In a regular Cayley map, the orientation-preserving map automorphism group acts regularly on darts. An exponent of a Cayley map is a number e with the property that, roughly speaking, the Cayley map is isomorphic to its 'e-fold rotational image'.

We establish necessary and sufficient conditions for a number e to be an exponent of a Cayley map and present results related to the question of existence of regular Cayley maps with given exponents and given distribution of inverses.

Summable graph labelings

Mirka Miller

A *labeling* (or *valuation*) of a graph is any map that carries some set of graph elements to numbers (usually, but not always, to the positive or non-negative integers). If the domain is the vertex set, the edge set, or the set of all vertices and edges, then the labeling is called respectively vertex labeling, edge labeling, or total labeling.

Additionally, we usually associate with a labeling an *evaluation* of labels incident to either each edge or each vertex. The evaluation is often a simple summation of the incident labels; the resulting value is often referred to as the *edge-* or *vertex-weight*. Various types of graph labelings can then be defined on the basis of the properties of the weights. For example, an *edge-magic labeling* requires that the edge-weights be all the same, an *antimagic labeling* requires that the weights be all different. A *sum labeling* requires that the vertex weights be themselves elements of the set of labels used in the graph.

More precisely, a *sum graph* is a graph G whose vertices are labeled with distinct integers in such a way that every pair of vertices labeled i and j is joined by an edge if and only if a vertex labeled $i + j$ also occurs in G.

Sum graphs were introduced by Harary in 1990. Our interest in sum graphs stems from the observation that the structure of a sum graph (that is, the arrangement of vertices and edges) is determined entirely by the labels assigned to its vertices: any two vertices i and j are joined by an edge if and only if vertex $i + j$ occurs in the graph. Thus every sequence of integers represents a sum graph. On the other hand, any connected graph G can be turned into a sum graph by the addition of isolated vertices. The minimum number of isolated vertices required to convert G into a sum graph G' is called the *sum number* $\sigma(G)$: G is then said to be *optimally labeled*.

In this talk we will give an overview of the state of our knowledge of sum graphs and various variations on the theme of sum graph labeling, including integral sum graph labeling, mod sum graph labeling, sum* graph labeling, and exclusive sum graph labeling. We will also consider various optimality issues for summable graph labelings. The paper concludes with a list of open problems in the area.

On the interval function of a connected graph

Ladislav Nebeský

Let G be a connected (finite or infinite) graph. If u and v are vertices of G, then by the interval between u and v we mean the set of all vertices t of G such that $d(u,t)+d(t,v) = d(u,v)$, where d denotes the distance function of G. By the interval function of G we mean the mapping I of $V(G)xV(G)$ into the set of all subsets of $V(G)$ such that $I(x,y)$ is the interval between x and y (for all vertices x and y in G). The subjects of the paper: graphs defined by their interval functions; mappings similar to the interval function; a characterization of the interval function.

Jakub Matěj Votický

Ladislav Nebeský

Votickho pozoruhodnou domnnku o barevnosti grafu zn dnes ji mlokdo. Pednka si klade za cl dlo tohoto neprvem zapomenutho prkopnka teorie graf oivit.

Variations on hamiltonian themes

Tomáš Kaiser, Daniel Král', Moshe Rosenfeld, Zdeněk Ryjáček and Heinz-Jürgen Voss

The hunt for Hamilton Cycles in graphs started in the 18-th century. Most probably Euler's search for a knight tour of the chess board was the first Hamilton Cycle problem considered. Nowadays we are flooded by theorems, conjectures, refuted conjectures, web sites whose common characteristic is that if a graph satisfies P then it is Hamiltonian. It is well known that the problem of finding a Hamilton Cycle in a graph is "certifiably" difficult. As common among Mathematicians, when the going gets rough, the rough "change" the problem. Current trends are to "refine" (mutilate) the definition of Hamilton Cycle. The first very minimal modification was to replace Hamilton Cycle by Hamilton Path. A more drastic modification is to allow vertices to be visited more than once. A 2-walk is a closed walk in a graph in which every vertex is visited but not more than twice. A 3-tree is a tree with maximum degree 3. Clearly, if G is Hamiltonian, it has a 2-walk but not vice versa. Thus, instead of searching for Hamiltonian cycles, several researchers, like M. Ellingham, N. Wormald, F. Ruskey and others, proposed searching for 2-walks, or even k-walks. These modifications create a hierarchy among graph properties:

$$\text{Hamiltonian} \subset \text{Traceable} \subset \text{2-walk} \subset \text{3-tree} \subset \text{3-walk} \ldots$$

In our work, we chose to retain the Hamilton Cycle and modify the graph. Instead of searching for Hamilton Cycles in G we look for Hamilton Cycles in the prism over G. This property is "sandwiched" between "Hamiltonian" and 2-walk. With this in mind, we revisit many theorems, conjectures, complexity problems etc. and test them for Hamilton Cycles in the prism over G.

For instance, in the 19-th century it was "believed" that all cubic, 3-connected planar graphs are Hamiltonian. In 1946, Tutte constructed the first counter example (with 46 vertices...). Is it true that the prisms over these graphs are Hamiltonian? YES! On the other hand, Nash-Williams conjectured that all 4-connected, 4-regular graphs are Hamiltonian. Infinitely many counter examples are known today. Such graphs clearly have a 2-walk. Do they have a Hamiltonian Prism? The complexity of determining whether a 4-connected, 4-regular graph is Hamiltonian is in NPC. On the other hand, a 2-walk can be constructed in Polynomial time. What about a Hamiltonian Cycle in the Prism?

In this talk I'll survey a sample of problems of this nature, and recent results.

The eccentric digraph of a (di)graph

Joe Ryan

The Eccentric Digraph of a graph G, ED(G), is a digraph on the same vertex set as G with an arc from u to v if and only if v is an eccentric vertex of u in G. Many properties of graphs and digraphs such as regularity, diameter, connectedness and degree sequence have been observed to impact on the structure of the eccentric digraphs. We conclude with some open problems.

On local and global independence numbers of a graph

Ralph J. Faudree, Zdeněk Ryjáček and Richard H. Schlep

The local independence number $\alpha_i(G)$ of a graph G at a distance i is the maximum number of independent vertices at distance i from any vertex. We study the impact of restricting $\alpha_i(G)$ on the (global) independence number $\alpha(G)$. Among others, we show that in graphs with bounded diameter, $\alpha(G)$ is bounded if and only if $\alpha_i(G)$ is bounded for at least one i, $2 \leq i \leq (\mathrm{diam}(G) - 1)/4$.

On supermagic and magic line graphs

Jaroslav Ivančo, Zuzana Lastivková and Andrea Semaničová

A graph is called *magic* (*supermagic*) if it admits a labeling of its edges by pairwise different (consecutive) positive integers such that the sum of the labels of the edges incident with a vertex is independent on the particular vertex. We characterize *magic line graphs* of general graphs, showing that the line graph $L(G)$ of connected graph G of size at least 5 is magic if and only if G does not belong to four forbidden families of graphs. Moreover, we deal with particular classes of *supermagic line graphs* of bipartite graphs. We prove that if G is bipartite regular graph of degree at least 3 then the line graph $L(G)$ is supermagic.

Additive induced-hereditary properties and unique factorization

Grzegorz Arkit, Alastair Farrugia, Peter Mihók, Gabriel Semanišin and Roman Vasky

A *graph property* is an isomorphism closed subset of the set of all graphs. A property $\mathcal{P}$ is $\preceq$*-hereditary* if, whenever G belongs to $\mathcal{P}$ and $H \preceq G$, then H belongs to $\mathcal{P}$ as well. A property that is closed under disjoint union of graphs is said to be *additive*.

The language of graph properties can be successfully used to generalise ordinary vertex colouring. In a *proper colouring* each colour class must be an independent set. In so-called *generalised colouring*, each colour class must have a prescribed graph property. Given a list of two or more properties, the class of all graphs that can be coloured according to that list is said to be a *reducible* property. One can immediately ask whether different lists correspond to different properties. This is the *unique factorisation problem*, which was solved affirmatively for additive hereditary and additive induced-hereditary graph properties.

We extend these results to induced-hereditary properties of directed coloured hypergraphs. Moreover we show that our result can be generalised beyond graphs and hypergraphs to other combinatorial objects such as oriented graphs, or partially ordered sets.

On the chromatic number of integer distance graphs

Roman Soták

Let D be a subset of the positive integers $\mathbf{N}$. The *integer distance graph* $G(\mathbf{Z}, D) = G(D)$ is defined as the graph with vertex set $V(G(D)) = \mathbf{Z}$ (the set of integers), and the edge set containing all pairs uv whose distance $|u - v| \in D$. D is called *distance set*. For the chromatic number of $G(D)$ we write $\chi(G(D)) = \chi(D)$. General bound for the chromatic number is $\chi(D) \leq |D| + 1$. We discuss two problems:

Problem 1: To find all sets D with $\chi(D) = |D| + 1$.

Problem 2: To find maximal value of $def(D) = \min\limits_{d \in D}\{\chi(D) - \chi(D - \{d\})\}$.

If d is an arbitrary divisor of the elements $d_1, d_2, \ldots, d_k$ of the distance set D then the integer distance graph $G(D) = G(d_1, d_2, \ldots, d_k)$ is isomorphic to d disjoint copies of $G(\frac{d_1}{d}, \frac{d_2}{d}, \ldots, \frac{d_k}{d})$. Hence next we will consider integer distance graphs $G(D)$ such that the greatest common divisor of the distance set elements is 1.

All known cases for problem 1 are: D with $|D| \leq 1$; $D = \{x, y\}$ with $x \equiv y \pmod 2$; $D = \{x, y, x + y\}$ with $x \not\equiv y \pmod 3$; $D = \{x, y, |x - y|, x + y\}$ with $x \equiv y \pmod 2$; $D = \{1, 4, 5, 6, 7\}$; $D = \{1, 2, \ldots, n - 1, kn\}$; $D = \{1, 2, \ldots, 2n - 1, 2n + 1, 4n\}$.

As for the problem 2 the only known fact is that there exist sets D with $def(D) = 2$, for instance the set $D = \{2, 5, 8, 9, 10\}$.

Edge-disjoint cycles through prescribed vertices

Luis Goddyn and Ladislav Stacho

We present a sufficient condition for the existence of edge-disjoint cycles, each containing a prescribed set of vertices in a graph. This generalizes several previously known Ore-type results. More precisely, we have the following theorem.

Let W be a subset of the vertices of a graph G of order n, and let $k \geq 0$ be an integer. Suppose the induced graph $G[W]$ is $2(k + 1)$-connected, and that for any two vertices having distance two in $G[W]$, at least one of the two has degree at least $n/2 + 2k$ in G. Then G contains $k + 1$ pairwise edge-disjoint cycles where each cycle contains all vertices in W.

A main lemma is a Hamilton-cycle packing result regarding complements of bipartite graphs: Let G be a $2k$-connected graph and let $G[X]$ and $G[Y]$ be cliques for some partition $V(G) = 3DX \cup Y$. Then G has k Hamilton cycles $C_1, \ldots, C_k$ such that $E(C_i) \cap E(C_j) \subseteq E(G[Y])$ for $i \neq 3Dj$. This lemma may be of great interest on its own; if $G = 3DK_{2k+1}$, and Y is a single vertex, then it becomes the well-know result on decomposition of K_{2k+1} into k edge-disjoint Hamilton cycles.

The sum labeling for fan

Kiki Ariyanti Sugeng

Let $G = (V, E)$ be simple undirected graph. The function L from V to a set of positive integers is called *sum graph labeling* of G if for distinct vertices $u, v \in V$ the edge $(u, v) \in E$ if and only if there is a vertex $w \in V$ such that $L(w) = L(u) + L(v)$.
A vertex $v \in V(G)$ is called a *working vertex* if $L(v)$ is the sum of two other distinct vertex labels. The labeling L is called *exclusive sum graph labeling* of G if there are no working vertices in G except the isolated vertices; Otherwise L is called an *inclusive sum graph labeling* of G.
Every graph can be labeled exclusively, but not every graph can be labeled inclusively. For example, wheel E_n for n odd can only have exclusive sum graph labeling, while for n even a wheel can be labeled exclusively or inclusively.
In this talk, we will give further examples of inclusive and exclusive sum graph labeling and present some new results obtained by our research group.

Light stars in large polyhedral maps

Milan Tuhársky

It is well known that every planar graph contains a vertex of degree at most 5. We investigate the existence of subgraphs H of bounded degree sum $w_G(H)$ of their vertices called the weight of H in G in polyhedral maps on 2-manifolds $\mathbf{M}$ of Euler characteristics $\chi(\mathbf{M}) \leq \mathbf{0}$; in particular, we study the case when H is a star.

Extremal questions for geometric graphs

Pavel Valtr

Geometric graphs are graphs drawn in the plane so that vertices are represented by distinct points in general position and edges are represented by segments connecting corresponding pairs of points.

We will discuss mainly Turán-type questions, e.g.: What is the maximum number of edges in a geometric graph on n vertices with no k pairwise disjoint (pairwise crossing, resp.) edges? We will also mention analogous questions for convex geometric graphs (a geometric graph is *convex* if its vertex set is formed by the vertices of a convex polygon).

On k-trestles in polyhedral graphs

Tomáš Kaiser, Michal Tkáč and Heinz-Jürgen Voss

A k-trestle of a graph G is a 2-connected spanning subgraph of G of maximum degree at most k. A graph is said to be chordal, if each cycle different from a 3-cycle has a chord. The toughness of a non-complete graph G is $\tau(G) = min(\frac{|S|}{\omega(G \setminus S)})$, where the minimum is to be taken over all nonempty separating vertex sets S, for which the number of components $\omega(G \setminus S)$ of $G \setminus S$ is at least 2. For the complete graph K_n let $\tau(K_n) = \infty$. Recently we have proved that for every integer k, $2 \leq k \leq 6$, each chordal polyhedral graph G of toughness greater than $\frac{3+2^{k-2}}{2+2^{k-1}}$ has a k-trestle. There are chordal polyhedral graphs of toughness $\frac{21}{29}$, or $\frac{51}{86}$, or $\frac{139}{259}$ without a 3-, or 4-, or 5-trestle, respectively. Here we prove that the toughness bounds are the same under the weaker assumption that the separating cycles of length at least four have chords.

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Program of the conference

MONDAY AFTERNOON

15:30	Ladislav Nebeský	On the interval function of a connected graph
16:20	**Break**	
16:40	Milan Tuhársky	Light stars in large polyhedral maps
17:05	Joe Ryan	The eccentric digraph of a (di)graph
17:30	Pavel Hrnčiar	Eccentric sequences and cycles in graphs
17:55	Gabriel Semanišin	Additive induced-hereditary properties and unique factorization
18:20	Petr Kolman	A note on the greedy algorithm for the unsplittable flow problem

TUESDAY MORNING

9:00	Pavel Valtr	Extremal questions for geometric graphs
9:50	**Break**	
10:10	Rostislav Caha	Paths and cycles with prescribed edges in hypercubes
10:35	Zuzana Kocková	Decomposition of even graph into closed trail
11:00	Ladislav Stacho	Edge-disjoint cycles through prescribed vertices
11:25	Roman Čada	Short disjoint paths in locally connected graphs
11:50	Yuqing Lin	On the edge connectivity of cages

TUESDAY AFTERNOON

16:30	Stanislav Jendroľ	Local properties of graphs embedded into surfaces
17:20	**Break**	
17:40	Mária Ipolyiová	The minimal polynomial of $2\cos(\pi/p)$ and regular maps of large planar width
18:05	Ľubica Líšková	Cayley maps and their exponents
18:30	Mirko Horňák	Neighbours distinguishing index of plane bipartite graphs

WEDNESDAY MORNING

9:00	Moshe Rosenfeld	Variations on hamiltonian themes
9:50	**Break**	
10:10	Roman Kužel	Every 4-connected line graph in hamiltonian if and only if it is hamiltonian connected
10:35	Martin Klazar	Power series in combinatorial enumeration

WEDNESDAY AFTERNOON

12:30	**Trip**	Castle Velhartice, Vydra river

THURSDAY MORNING

9:00	Mirka Miller	Summable graph labelings
9:50	**Break**	
10:10	Kiki A. Sugeng	Inclusive and Exclusive Sum Graph Labeling
10:35	Andrea Semaničová	On the magic and supermagic line graphs
11:00	Dalibor Fronček	Vertex magic total labelings of products of graphs
11:25	Michael Kubesa	α-like labeling and drafting of the graphs
11:50	Martin Bača	Labelings of graphs

THURSDAY AFTERNOON

16:00	Daniel Král	Coloring powers of chordal graphs
16:25	Roman Soták	On the chromatic number of integer distance graphs
16:50	**Break**	
17:10	Jiří Fiala	Generalized H-coloring and H-covering of trees
17:35	Martin Knor	Face 2-colorable triangulations of $K_{n,n,n}$ and biembeddings of Latin squares
18:00	**Problem session**	

FRIDAY MORNING

10:00	Tomáš Kaiser	2-walks in graphs
10:25	Erik Bruoth	On-line ranking of dodecahedron
10:50	**Break**	
11:10	Jan Kratochvíl	Geometric intersection graphs
11:35	Zdeněk Ryjáček	On local and global independence numbers of a graph

The organisers are grateful to the **Platan** *brewery for sponsoring.*