Contractible Subgraphs, Thomassen's Conjecture and the Dominating Cycle Conjecture for Snarks

Hajo Broersma [1]
Gašper Fijavž [2]
Tomáš Kaiser [3,4]
Roman Kužel [3,4]
Zdeněk Ryjáček [3,4]

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Abstract

We show that the conjectures by Matthews and Sumner (every 4-connected claw-free graph is hamiltonian), by Thomassen (every 4-connected line hraph is hamiltonian) and by Fleischner (every cyclically 4-edge-connected cubic graph has either a 3-edge-coloring or a dominating cycle), which are known to be equivalent, are equivalent with the statement that every snark (i.e. a cyclically 4-edge-connected cubic graph of girth at least five that is not 3-edge-colorable) has a dominating cycle.

We use a refinement of the contractibility technique which was introduced by the last author and R.H. Schelp as a common generalization and strengthening of the reduction techniques by Catlin and Veldman and of the closure concept introduced by the last author.

Keywords: dominating cycle, contractible graph, cubic graph, snark, line graph, hamiltonian graph

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[1]Department of Computer Science, University of Durham, Science Laboratories, South Road, Durham, DH1 3LE England, e-mail `hajo.broersma@durham.ac.uk`.

[2]Faculty of Computer and Information Science, University of Ljubljana, Tržaška 25, 1000 Ljubljana, Slovenia, e-mail `gasper.fijavz@fri.uni-lj.si`.

[3]Department of Mathematics, University of West Bohemia, and Institute for Theoretical Computer Science (ITI), Charles University, P.O. Box 314, 306 14 Pilsen, Czech Republic, e-mail {`kaisert,rkuzel,ryjacek`}`@kma.zcu.cz`.

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1 Introduction

In this paper we consider finite loopless undirected graphs. However, we allow the graphs to have multiple edges. We follow the most common graph-theoretic terminology and notation and for concepts and notation not defined here we refer the reader to [2]. Unlike in [2], the induced subgraph of a graph G on a set of vertices $M \subset V(G)$ is denoted $\langle M \rangle_G$. A graph G is *claw-free* if G does not contain an induced copy of the claw $K_{1,3}$.

In 1984, Matthews and Sumner [10] posed the following conjecture.

Conjecture A [10]. *Every 4-connected claw-free graph is hamiltonian.*

Since every line graph is claw-free (see [1]), the following conjecture by Thomassen is a special case of Conjecture A.

Conjecture B [13]. *Every 4-connected line graph is hamiltonian.*

A closed trail T in a graph G is said to be *dominating*, if every edge of G has at least one vertex on T, i.e., the graph $\langle V(G) \setminus V(T) \rangle_G$ is edgeless (a closed trail is defined as usual, except that we allow a single vertex to be such a trail). The following result by Harary and Nash-Williams [7] shows the relation between the existence of a dominating closed trail (abbreviated DCT) in a graph and hamiltonicity of its line graph.

Theorem 1 [7]. *Let G be a graph with at least three edges. Then $L(G)$ is hamiltonian if and only if G contains a DCT.*

For an integer k, a graph G with $|E(G)| > k$ is said to be *essentially k-edge-connected* if G contains no edge-cut R such that $|R| < k$ and at least two components of $G - R$ are nontrivial (i.e. containing at least one edge). If G contains no edge-cut R such that $|R| < k$ and at least two components of $G - R$ contain a cycle, G is said to be *cyclically k-edge-connected.*

It is well-known that G is essentially k-edge-connected if and only if its line graph $L(G)$ is k-connected. Thus, the following statement is an equivalent formulation of Conjecture B.

Conjecture C. *Every essentially 4-edge-connected graph contains a DCT.*

Specifically, if G is *cubic* (i.e. regular of degree 3), then a DCT becomes a dominating cycle (abbreviated DC). Since a cubic graph is essentially 4-edge-connected if and only if it is cyclically 4-edge-connected (see [6]), the following statement, known as the Dominating Cycle Conjecture, is a special case of Conjecture C.

Conjecture D. *Every cyclically 4-edge-connected cubic graph has a dominating cycle.*

Restricting to cyclically 4-edge-connected cubic graphs that are not 3-edge-colorable, we obtain the following conjecture posed by Fleischner [5].

Conjecture E [5]. *Every cyclically 4-edge-connected cubic graph that is not 3-edge-colorable has a dominating cycle.*

In [11], a closure technique was used to prove that Conjectures A and B are equivalent. Fleischner and Jackson [6] showed that Conjectures B, C and D are equivalent. Finally, Kochol [8] established the equivalence of these conjectures with Conjecture E. Thus, we have the following result.

Theorem 2 [6], [8], [11]. *Conjectures A, B, C, D and E are equivalent.*

Note that recently Kužel and Xiong [9] showed the equivalence of these conjectures with the statement that every 4-connected line graph is hamiltonian-connected.

A cyclically 4-edge-connected cubic graph G of girth $g(G) \geq 5$ that is not 3-edge-colorable is called a *snark*. Restricting our considerations to snarks, we obtain the following special case of Conjecture E.

Conjecture F. *Every snark has a dominating cycle.*

In the main result of this paper, Theorem 11, we show that Conjecture F is equivalent with the previous ones.

Note that it is easy to observe that every cyclically 4-edge-connected cubic graph other than K_4 must be triangle-free. Thus, the difference between Conjectures E and F consists in restricting to graphs which do not contain a 4-cycle. For the proof of the equivalence of these conjectures we develop a refinement of the technique of contractible subgraphs that was developed in [12] as a common generalization of the closure concept [11] and Catlin's collapsibility technique [3].

2 Weakly contractible graphs

In this section we introduce a refinement of the contractibility technique under a special assumption which is automatically satisfied in cubic graphs. We basically follow the terminology and notation of [12].

For a graph H and a subgraph $F \subset H$, $H|_F$ denotes the graph obtained from H by identifying the vertices of F as a (new) vertex v_F, and by replacing the created loops by pendant edges ($H|_F$ may contain multiple edges). For a subset $X \subset V(H)$ and a partition $\mathcal{A}$ of X into subsets, $E(\mathcal{A})$ denotes the set of all edges a_1a_2 (not necessarily in H) such that a_1 and a_2 are in the same element of $\mathcal{A}$, and $H^{\mathcal{A}}$ denotes the graph with vertex set $V(H^{\mathcal{A}}) = V(H)$ and edge set $E(H^{\mathcal{A}}) = E(H) \cup E(\mathcal{A})$ (here the sets $E(H)$ and $E(\mathcal{A})$ are considered to be disjoint, i.e. if $e_1 = a_1a_2 \in E(H)$ and $e_2 = a_1a_2 \in E(\mathcal{A})$, then e_1, e_2 are parallel edges in $H^{\mathcal{A}}$).

Let F be a graph and $A \subset V(F)$. Then F is said to be A-*contractible*, if for every even subset $X \subset A$ and for every partition $\mathcal{A}$ of X into two-element subsets, the graph $F^{\mathcal{A}}$ has a DCT containing all vertices of A and all edges of $E(\mathcal{A})$ (specifically, if $X = \emptyset$, then $F^{\mathcal{A}} = F$).

If H is a graph and $F \subset H$, then a vertex $x \in V(F)$ is said to be a *vertex of attachment of F in H* if x has a neighbor in $V(H) \setminus V(F)$. The set of all vertices of attachment of F in H is denoted by $A_H(F)$. Finally, $d_T(H)$ denotes the maximum number of edges of a graph H that are dominated by a closed trail T in H (i.e., H has a DCT if and only if $d_T(H) = |E(H)|$).

The following was proved in [12].

Theorem 3 [12]. *Let F be a connected graph and let $A \subset V(F)$. Then F is A-contractible if and only if*

$$d_T(H) = d_T(H|_F)$$

for every graph H such that $F \subset H$ and $A_H(F) = A$.

Specifically, F is A-contractible if and only if, for any H such that $F \subset H$ and $A_H(F) = A$, H has a DCT if and only if $H|_F$ has a DCT.

Let F be a graph of minimum degree $\delta(F) \geq 2$ and let $A \subset V(F)$. The graph F is said to be *weakly A-contractible*, if for every nonempty even subset $X \subset A$ and for every partition $\mathcal{A}$ of X into two-element subsets, the graph $F^{\mathcal{A}}$ has a DCT containing all vertices of A and all edges of $E(\mathcal{A})$.

Thus, in comparison with the contractibility concept as introduced in [12], we do not consider the case $X = \emptyset$. Since obviously every A-contractible graph F satisfies $\delta(F) \geq 2$, every A-contractible graph is also weakly A-contractible.

We show that, in a special situation, weak contractibility is sufficient to obtain the equivalence of Theorem 3.

Theorem 4. *Let F be a connected graph and let $A \subset V(F)$. Then F is weakly A-contractible if and only if*

$$d_T(H) = d_T(H|_F)$$

for every graph H such that $F \subset H$, $A_H(F) = A$, $d_{H-F}(a) = 1$ for every $a \in A$, and at least one vertex of A is not a cutvertex of H.

Proof. The proof of Theorem 4 basically follows the proof of Theorem 2.1 of [12].

If $F \subset H$, then every closed trail T in H corresponds to the closed trail $T|_F$ in $H|_F$, dominating at least as many edges as T. Hence immediately $d_T(H) \leq d_T(H|_F)$.

Suppose that F is weakly A-contractible and let T' be a closed trail in $H|_F$ dominating maximum number of edges. If $v_F \notin V(T')$, then T' is also a closed trail in H, implying $d_T(H|_F) \leq d_T(H)$, as requested. Hence suppose $v_F \in V(T')$. Since not every vertex of A is a cutvertex of H, T' is nontrivial. Then the edges of T' determine in H a system of trails $\mathcal{P} = \{P_1, \ldots, P_k\}$ such that every $P_i \in \mathcal{P}$ has endvertices in A (note that all trails in $\mathcal{P}$ are open since $d_{H-F}(a) = 1$ for all $a \in A$). Clearly, every $x \in A$ is an endvertex of at most one trail from $\mathcal{P}$.

Set $X = \{x \in A_H(F) | \ x$ is an endvertex of some $P_i \in \mathcal{P}\}$ and $\mathcal{A} = \{A_1, \dots, A_k\}$, where A_i is the (two-element) set of endvertices of P_i, $i = 1, \dots, k$. Since $v_F \in V(T')$, the set X is nonempty.

By the weak A-contractibility of F, $F^{\mathcal{A}}$ has a DCT Q, containing all vertices of A and all edges of $E(\mathcal{A})$. The trail Q determines in F a system of trails $Q_1, \dots, Q_k$ such that every Q_i has its two endvertices in two different elements of $\mathcal{A}$. Now, the trails Q_i together with the system $\mathcal{P}$ form a closed trail in H, dominating at least as many edges as T'. Hence $d_T(H|_F) \le d_T(H)$, implying $d_T(H|_F) = d_T(H)$.

Next suppose that F is not weakly A-contractible. Then, for some nonempty $X \subset A$ and a partition $\mathcal{A}$ of X into two-element sets, $F^{\mathcal{A}}$ has no DCT containing all vertices of A and all edges of $E(\mathcal{A})$. Let $\mathcal{A} = \{\{x_1', x_1''\}, \dots, \{x_k', x_k''\}\}$, and construct a graph H by joining k vertex disjoint x_i', x_i''-paths P_i of length at least 3, $i = 1, \dots, k$, to the vertices of X, and by attaching a pendant edge to every vertex in $A \setminus X$. Since F is not weakly contractible, H has no DCT. Since clearly $H|_F$ has a DCT, we have $d_T(H) < d_T(H|_F)$. ■

In the special case of cubic graphs, we have the following corollary.

Corollary 5. *Let F be a connected graph with $\delta(F) = 2$ and $\Delta(F) = 3$ and let $A = \{x \in V(F) \mid d_F(x) = 2\}$. Then F is weakly A-contractible if and only if*

$$d_T(H) = d_T(H|_F)$$

for every cubic graph H such that $F \subset H$, $A_H(F) = A$, and at least one vertex of A is not a cutvertex of H.

Proof. Clearly $d_{H-F} = 1$ for every $a \in A$, since H is cubic. If F is weakly A-contractible, then $d_T(H) = d_T(H|_F)$ immediately by Theorem 4. For the rest of the proof, it is sufficient to modify the last part of the proof of Theorem 4 such that the constructed graph H is cubic. To achieve this, it is sufficient to use a copy of the graph in Figure 1(a) instead of each of the paths P_i, and a copy of the graph in Figure 1(b) instead of each of the pendant edges attached to the vertices $a_j \in A \setminus X$. Then $H|_F$ has a closed trail dominating all the edges e_j while in H there is no such closed trail. ■

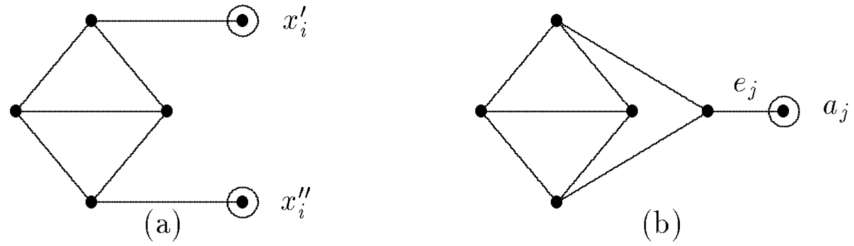

Figure 1

We say that a subgraph $F \subset H$ is a *weakly contractible subgraph of* H if F is weakly $A_H(F)$-contractible. We then have the following corollary.

Corollary 6. *Let H be a cubic graph and let F be a weakly contractible subgraph of H. Then H has a dominating cycle if and only if $H|_F$ has a DCT.*

Proof. First note that in a cubic graph every closed trail is a cycle. Since H is cubic and $\delta(F) \geq 2$, $A_H(F) = \{x \in V(F) \mid d_F(x) = 2\}$. Rest of the proof follows immediately from Corollary 5. ■

Examples. 1. The graphs in Figure 2 are examples of graphs that are weakly A-contractible but not A-contractible (vertices of the set A are double-circled).

2. As shown in [3], the triangle C_3 is collapsible, and hence C_3 is also A-contractible for any subset A of its vertex set.

3. Let C be a cycle of length $\ell \geq 4$, let $x, y \in V(C)$ be nonadjacent and set $A = V(C)$, $X = \{x, y\}$ and $\mathcal{A} = \{\{x, y\}\}$. Then there is no DCT in C containing the edge $xy \in C^{\mathcal{A}}$ and all vertices of A. Hence no cycle C of length at least 4 is weakly $V(C)$-contractible.

In Section 3 we will develop a technique that allows to handle large cycles by replacing them with another suitable non-contractible graph. Note that an alternative attempt to refine the collapsibility technique allowing to (partially) handle cycles of length 4 was done in [4].

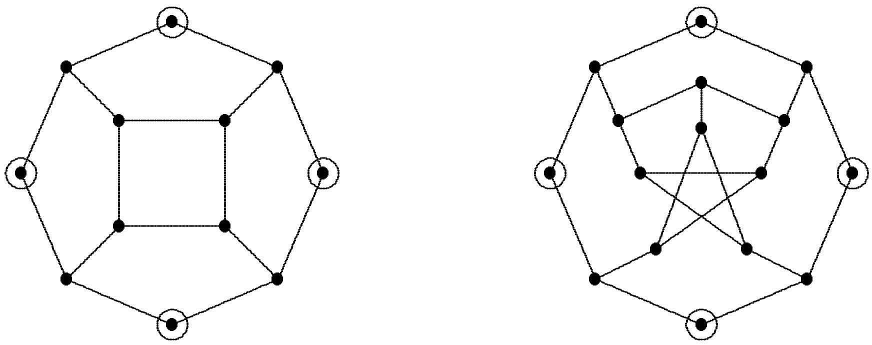

Figure 2

We conjecture the following.

Conjecture G. *Every cyclically 4-edge-connected cubic graph contains a weakly contractible subgraph.*

Theorem 7. *Conjecture G is equivalent with Conjectures A, B, C, D and E.*

Proof. We first show that Conjecture G implies Conjecture D. Thus, suppose Conjecture G is true and let G be a minimum counterexample to Conjecture D. Let $F \subset G$ be a weakly contractible subgraph of G and set $A = A_G(F)$, $t = |A| \geq 4$. By Corollary 6, the graph $G|_F$ has no DCT.

We use the following operation (see [6]). Let H be a graph, let $v \in V(H)$ be of degree $d = d_H(v) \geq 4$, and let $x_1, \ldots, x_d$ be an ordering of the neighbors of v (allowing repetition in case of multiple edges). Let H' be the graph obtained by adding edges x_iy_i, $i = 1, \ldots, d$, to the disjoint union of the graph $H - v$ and the cycle $y_1y_2 \ldots y_dy_1$. Then H' is said to be an *inflation of H at v*. The following fact was proved in [6].

Claim [6]. *Let H be an essentially 4-edge-connected graph of minimum degree $\delta(G) \geq 3$ and let $v \in V(H)$ be of degree $d(v) \geq 4$. Then some inflation of H at v is essentially 4-edge-connected.*

Now let G' be an essentially 4-edge-connected inflation of $G|_F$ at v_F. Then G' is a cubic graph having no DC (since otherwise $G|_F$ would have a DCT). Since no cycle of length $\ell \geq 4$ is weakly contractible, F is not a cycle. But then $|E(G')| < |E(G)|$, contradicting the minimality of G.

For the rest of the proof, it is sufficient to show that Conjecture D implies Conjecture G. Indeed, if C is a dominating cycle in G, $e = uv \in E(C)$ and $A = \{u, v\}$, then the graph $F = G - e$ is a weakly A-contractible subgraph of G. ∎

It should be noted here that the second part of the proof of Theorem 7 is based on a construction with $|A| = 2$, which forces $G - F$ to be trivial since G is cyclically 4-edge-connected. It is straightforward to observe that the following stronger statement implies Conjectures A – G. However, we do not know whether these statements are equivalent.

Conjecture H. *Every cyclically 4-edge-connected cubic graph G contains a weakly contractible subgraph F with $|A_G(F)| \geq 4$.*

3 Replacement of a subgraph

Let G be a graph, $F \subset G$, and let G_{-F} be the graph with vertex set $V(G_{-F}) = V(G) \setminus (V(F) \setminus A_G(F))$ and with edge set $E(G_{-F}) = E(G) \setminus \{xy \in E(G) \mid x, y \in V(F)\}$ (i.e. G_{-F} is obtained from G by removing all non-attachment vertices of F and all edges with both vertices in $V(F)$). Let F' be a graph such that $V(F') \cap V(G) = \emptyset$, let $A' \subset V(F')$ be such that $|A'| = |A_G(F)|$ and let $\varphi : A_G(F) \to A'$ be a one-to-one mapping. Let H be the graph obtained from G_{-F} by identifying each $x \in A_G(F)$ with its image $\varphi(x) \in A'$. We say that H is obtained by *replacement of F by F' modulo φ* and denote $H = G[F \stackrel{\varphi}{\to} F']$.

The following observation shows that the replacement of a weakly contractible subgraph by another one affects neither the existence nor the nonexistence of a DCT.

Proposition 8. *Let G be a graph and let $F \subset G$ be a weakly contractible subgraph of G such that $d_{G-F}(x) = 1$ for every $x \in A_G(F)$. Let F' be a weakly A'-contractible graph for an $A' \subset V(F')$, and let $\varphi : A_G(F) \to A'$ be a one-to-one mapping. Then G has a DCT if and only if the graph $H = G[F \stackrel{\varphi}{\to} F']$ has a DCT.*

Proof. By Theorem 4, G has a DCT if and only if $G|_F$ has a DCT. Similarly, H has a DCT if and only if $H|_{F'}$ has a DCT. But the graphs $G|_F$ and $H|_{F'}$ are, up to the number of pendant edges at v_F ($v_{F'}$), isomorphic. ■

In the special case of cubic graphs, we obtain the following consequence.

Corollary 9. *Let G be a cubic graph and let $F \subset G$ be a weakly contractible subgraph of G. Let F' be a graph with $\delta(F') = 2$ and $\Delta(F') = 3$, let $A' = \{x \in V(F') |\ d_{F'}(x) = 2\}$ and suppose that F' is weakly A'-contractible. Let $\varphi : A_G(F) \to A'$ be a one-to-one mapping. Then G has a DC if and only if the graph $H = G[F \overset{\varphi}{\to} F']$ has a DC.* ■

Let F be a graph, $A \subset V(F)$, let X be a nonempty even subset of A and let $\mathcal{A}$ be a partition of X into two-element subsets. Let $\mathcal{T}(\mathcal{A})$ denote the system of all closed trails in $F^{\mathcal{A}}$ containing all edges from $E(\mathcal{A})$. For a trail $T \in \mathcal{T}(\mathcal{A})$, we set $C(T) = \{x \in A \setminus X |\ x \in V(T)\}$. We say that $(X, \mathcal{A})$ is a *good pair* if there is a $T \in \mathcal{T}(\mathcal{A})$ with $C(T) = A \setminus X$.

Let F_1, F_2 be graphs, $A_i \subset V(F_i)$, $i = 1, 2$, and let $\varphi : A_1 \to A_2$ be a one-to-one mapping. For any $X \subset A_1$, we denote $\varphi(X) = \{\varphi(x) |\ x \in X\}$ and, for any partition $\mathcal{A}$ of X, we set $\varphi(\mathcal{A}) = \{\varphi(A_i) |\ A_i \in \mathcal{A}\}$. A mapping $\varphi : A_1 \to A_2$ is a *compatible mapping* if φ is a one-to-one mapping such that for any pair $(X, \mathcal{A})$ and $T \in \mathcal{T}(\mathcal{A})$ there is a trail $T' \in \mathcal{T}(\varphi(\mathcal{A}))$ such that $\varphi(C(T)) \subset C(T')$.

Note that a compatible mapping always maps a good pair on a good pair. Although a compatible mapping is one-to-one, the inverse φ^{-1} need not be compatible.

Example. Let F_1, F_2 be the graphs in Figure 3, $A_i = \{a_1^i, a_2^i, a_3^i, a_4^i\}$, $i = 1, 2$, and let $\varphi : A_1 \to A_2$ be the mapping that maps a_j^1 on a_j^2, $j = 1, 2, 3, 4$. Then φ is a compatible mapping. Note that there is no compatible mapping of A_2 onto A_1.

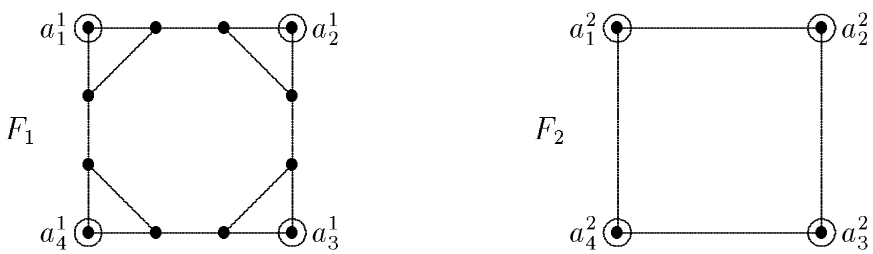

Figure 3

Theorem 10. *Let G be a graph having a DCT, let $F \subset G$ and suppose that $d_{G-F}(a) = 1$ for every $a \in A = A_G(F)$. Let F' be a graph, let $A' \subset V(F')$ and let $\varphi : A \to A'$ be a compatible mapping. Then the graph $H = G[F \overset{\varphi}{\to} F']$ has a DCT.*

(Note that if both φ and φ^{-1} are compatible, then G has a DCT if and only if $H = G[F \overset{\varphi}{\to} F']$ has a DCT.)

Proof. Let T be a DCT in G. Then the edges of T in $E(G) \setminus E(F)$ determine a nonempty even subset $X \subset A$ and a partition $\mathcal{A}$ of X into two-element subsets in a way similar to that in the proof of Theorem 4. Specifically, $\mathcal{T}(\mathcal{A}) \neq \emptyset$. By the compatibility of φ, there is a $T' \in \mathcal{T}(\varphi(\mathcal{A}))$ with $C(T') \supset \varphi(C(T))$. Then the edges of the set $(E(T) \cap (E(G) \setminus E(F)) \cup (E(T') \cap E(F'))$ determine a DCT in H. ■

Example. Let F_3 be the graph in Figure 4, set $A_3 = \{a_1^3, a_2^3, a_3^3, a_4^3\}$, and let F_2 and A_2 be as in the previous example. The graph F_3 has no DCT containing the edge $a_1^3a_3^3$ and both the vertices a_2^3, a_4^3, and symmetrically also no DCT containing the edge $a_2^3a_4^3$ and both the vertices a_1^3, a_3^3. Hence it is easy to check that $\varphi : A_3 \to A_2$ that maps a_j^3 on a_j^2, $j = 1, 2, 3, 4$, is a compatible mapping.

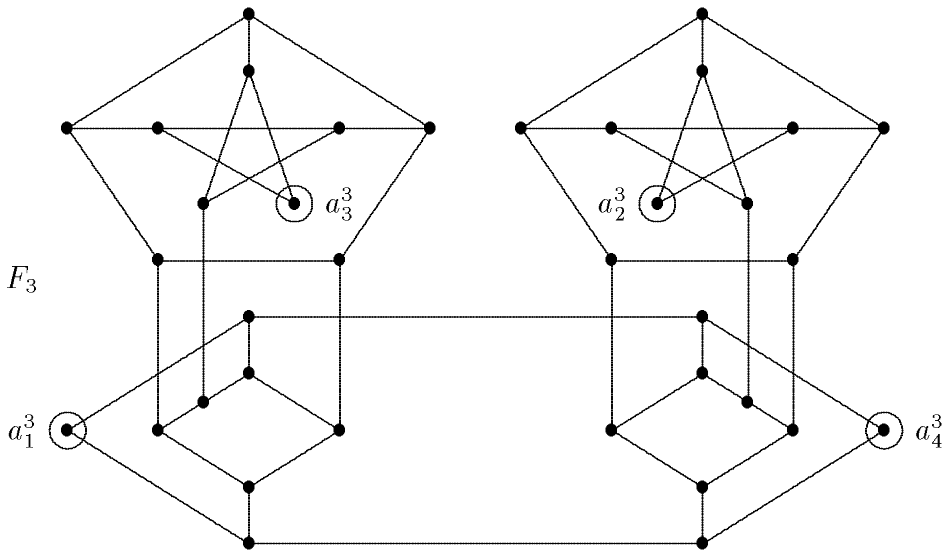

Figure 4

Now we are ready to prove that Conjectures A – E can be equivalently restricted to snarks.

Theorem 11. *Conjecture F is equivalent with Conjectures A, B, C, D and E.*

Proof. Clearly, Conjecture E implies Conjecture F. By Theorem 2, it is sufficient to show that Conjecture F implies Conjecture D. Thus, let G be a counterexample to Conjecture D, i.e. a cyclically 4-edge-connected cubic graph without DC. For any cycle C of length 4 in G, choose a compatible mapping of the graph F_3 of Figure 4 on C, and let G' be the graph obtained by recursively replacing every cycle of length 4 by a copy of F_3. Then G' is a cyclically 4-edge-connected cubic graph of girth $g(G') \geq 5$ and, by Theorem 10, G' has no DC. If G' is not 3-edge-colorable, G' is a snark and we are done. Otherwise, we use the following fact and construction by Kochol [8].

Claim [8]. *If a cubic graph G contains the graph H of Figure 5 as an induced subgraph, then G is not 3-edge-colorable.*

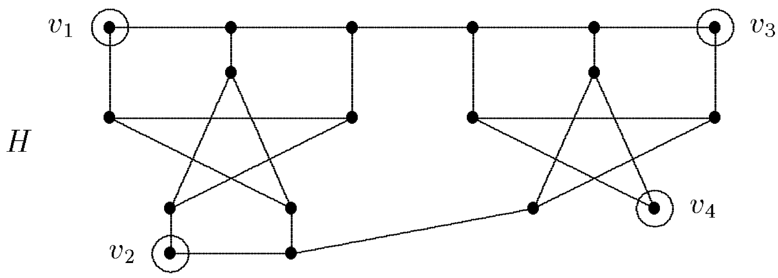

Figure 5

We use the claim as follows. Let $xy \in E(G)$, let x', x'' (y', y'') be the neighbors of x (of y) different from y (x), respectively, and let G_i', $i = 1, 2, 3$, be three copies of the graph $G - x - y$ (where x_i', x_i'', y_i', y_i'' are the copies of x', x'', y', y'' in G_i), $i = 1, 2, 3$. Then the graph $\bar{G}$ obtained from G_1, G_2, G_3 and H by adding the edges $x_1'v_3$, $x_1''v_4$, $y_1'x_2'$, $y_1''x_2''$, $y_2'x_3'$, $y_2''x_3''$, $y_3'v_1$ and $y_3''v_2$ is a cyclically 4-edge-connected graph of girth $g(\bar{G}) \geq 5$. By the claim, $\bar{G}$ is not 3-edge-colorable. It remains to show that $\bar{G}$ has no DC.

Let, to the contrary, C be a DC in $\bar{G}$. Then it is easy to check that for some $i \in \{1, 2, 3\}$, the intersection of C with G_i is either a path with one end in $\{x_i', x_i''\}$ and second in $\{y_i', y_i''\}$, or two such paths. But, in both cases, the path(s) can be easily extended to a DC in G, a contradiction. ■

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