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Tomáš Kaiser (ed.)

Graphs 2008
(Zadov, June 9–13, 2008)

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GRAPHS 2008

Preface

The 43rd Czech–Slovak Conference on Combinatorics and Graph Theory, **Graphs 2008**, is held on June 9–13, 2008, in Zadov. The conference is organized by

- Department of Mathematics, University of West Bohemia in Plzeň,
- the Plzeň branch of the Institute for Theoretical Computer Science (ITI),
- the Union of Czech Mathematicians and Physicists (JČMF), local chapter Plzeň, and
- Combinatorica, o. p. s.

Support from project 1M0545 and Research Plan MSM 4977751301 of the Czech Ministry of Education is gratefully acknowledged.

Information about the conference may be found at the web page

http://iti.zcu.cz/grafy08.

We hope that the participants will find the meeting in Zadov enjoyable and inspiring.

the Organizers

Programme

Conference programme

Monday, June 9

12:00– –14:00	Lunch
14:30	**Edita Máčajová: Cycles and matchings in cubic graphs**
15:20	Zdzisław Skupień: On the LOGO and the exponentiation of snarks
15:45	Robert Lukoťka: Snarks with given real flow number
16:10	Coffee break
16:40	Naďa Krivoňáková: On the $(k, n-k)$-hypomorphic graphs
17:05	Peter Czimmermann: On a certain algorithm for the graph isomorphism problem
17:30	Ján Mazák: Circular chromatic index of Blanuša snarks
17:55	Jan Kratochvíl: Generalized domination in degenerate graphs
19:00	Dinner/Welcome party

Tuesday, June 10

7:30	Breakfast
9:00	**Ingo Schiermeyer: Rainbow colourings**
9:50	Mária Ždímalová: Graph coverings in constructions of large graphs and digraphs with given diameter and degree
10:15	Coffee break
10:45	Štefan Gyürki: Goal-minimally k-diametric graphs
11:10	Marcel Abas: Isomorphism of reverse Cayley digraphs
11:35	Ľubica Staneková: Partially t-balanced Cayley maps
12:00	Martin Nehéz: On renormalization of scale-free networks
12:30	Lunch
15:00	**Daniel Kráľ: Cycle covers of graphs**
15:50	Robert Šámal: Spectra of (3,6)-fullerenes
16:15	Coffee break
16:45	Roman Čada: Inequalities for domination number
17:10	Petr Gregor: Perfect parallel-edge-dominating coloring of the hypercube and Turán-type problems
17:35	Jaroslav Opatrny: Local algorithms for Unit Disk graphs

18:00 Ľubomír Török: Antibandwidth of d-dimensional meshes
19:00 Dinner

Wednesday, June 11

7:30 Breakfast
9:00 **Petr Kovář: Spanning tree factorizations of complete graphs**
9:50 Dalibor Fronček: Decomposition of $K_{n,n}$ into $(0, j)$-prisms
10:15 Coffee break
10:45 Pavel Valtr: On embedding triconnected cubic graphs on point sets
11:10 Sergej Ševec: Structure of noninscribability
11:35 Martin Pergel: Recognition problem and sandwiching
12:00 Lunch

13:30 Excursion
19:00 Dinner

Thursday, June 12

7:30 Breakfast
9:00 **Tomáš Madaras: Theory of light graphs and related topics**
9:50 Mirko Horňák: Maximum weight of a graph of given order, size and property
10:15 Coffee break
10:45 Róbert Hajduk: On large light graphs in families of polyhedral graphs
11:10 Kristína Budajová: Parity vertex colouring of graphs
11:35 Július Czap: Colouring vertices of plane graphs under restrictions given by faces
12:00 Ondřej Rucký: Strong parity vertex coloring of outerplanar graphs
12:30 Lunch

15:00 **Petr Hliněný: Approaching tree-width of graphs from a matroidal perspective**
15:50 Ondrej Moriš: Branch-decomposition applied to route-planning

16:15 Coffee break
16:45 Ondřej Suchý: Some parameterized problems related to Seidel's switching
17:10 Robert Ganian: Automata formalization for graphs of bounded rank-width
17:35 Andrea Semaničová: On super (a,d)-edge-antimagic total labelings of disconnected graphs
18:00 Vladimír Vetchý: Distance regular square of distance regular graphs
19:00 Dinner

Friday, June 13

7:30 Breakfast
9:00 **Martin Knor: Face 2-colourable triangulations by $K_{n,n,n}$ and Latin squares**
9:50 Igor Fabrici: On vertices enforcing a hamiltonian cycle
10:15 Coffee break
10:45 Marek Tesař: Covering projection of regular graphs
11:10 Ruslan Gumerov: Hamiltonicity in $CN_{i,j,k}$-free graphs
11:35 Tomáš Kaiser: Hamilton cycles in star graphs
12:30 Lunch
14:00 Conference ends

Abstracts

Isomorphism of reverse Cayley digraphs

Marcel Abas

If $G = C(\Gamma, X)$ is a Cayley digraph for the group Γ and the generating set X, its *reverse* $G^{-1} = C(\Gamma, X^{-1})$ is the Cayley digraph for the group Γ and the generating set X^{-1}.

In this contribution we deal with isomorphisms of G and G^{-1} for some special metacyclic groups.

Parity vertex colouring of graphs

Kristína Budajová

(joint work with Stanislav Jendroľ and Stanislav Krajči)

Consider a graph $G = (V, E)$. Let $\varphi : V \to \{1, 2, ..., k\}$ be an assignment of colours from the set $\{1, 2, ..., k\}$ to the vertices of G. Such a colouring is called *vertex k-colouring.* Let a *parity path* in a vertex colouring of a graph G is a path along which each colour is used an even number of times. The vertex colouring of a graph G is called the *parity vertex colouring* if G has no parity path. The minimum k for which the graph G has the parity vertex k-colouring is called the *parity vertex chromatic number* of G and denoted by $\chi_p(G)$.

The main result of our paper is the following theorem:

Theorem. *Let T be a tree. Then*

$$\lceil \log_2(2 + diam(T)) \rceil \leq \chi_p(T) \leq 1 + rad(T).$$

Moreover, both bounds are tight.

Colouring vertices of plane graphs under restrictions given by faces

Július Czap

(joint work with Stanislav Jendroľ)

Consider a vertex colouring of a connected plane graph G. A colour c is used k times by a face α of G if it appears k times along the facial walk of α. Two natural problems arise.

(1) A vertex colouring φ is a *weak parity vertex colouring* of a connected plane graph G with respect to its faces (wpv colouring) if each face of G uses at least one colour an odd number of times. Problem is to determine the minimum number $\chi_w(G)$ of colours used in a wpv colouring of G.

(2) A vertex colouring φ is a *strong parity vertex colouring* of a 2-connected plane graph G with respect to the faces of G (spv colouring) if each face of G that uses a colour then it uses an odd number of times. Problem is to find the minimum number $\chi_s(G)$ of colours used in an spv colouring of G.

We have proved that $\chi_w(G) \leq 4$ for every connected plane graph with minimum face degree at least 3.

We present our other recent results and open questions concerning the above mentioned problems.

References

[CJ] J. Czap and S. Jendroľ, Colouring vertices of plane graphs under restrictions given by faces, *Discussiones Math. Graph Theory* (submitted).

On a certain algorithm for the graph isomorphism problem

Peter Czimmermann

The graph isomorphism problem is the problem to decide if two given graphs are isomorphic. In this contribution we will present a certain algorithm for the graph isomorphism problem which works with the number of walks between vertices. We will study the complexity and the efficiency of the algorithm. We also consider some connections between the properties of possible counterexamples to the reconstruction conjecture and the properties of graphs for which the algorithm fails.

Inequalities for domination number

Roman Čada

In 1963 Vizing posed the following conjecture (γ denotes the domination number of a graph and '$\Box$' the Cartesian product of graphs).

Conjecture. *For any graphs G and H, $\gamma(G)\gamma(H) \leq \gamma(G\Box H)$.*

An analogous conjecture involving the upper domination number Γ due to Nowakowski and Rall was proved recently by Brešar.

Theorem. *For any graphs G and H, $\Gamma(G)\Gamma(H) \leq \Gamma(G\Box H)$.*

A result in the direction of Vizing's conjecture due to Clark and Suen says that for any graphs G and H, $\gamma(G)\gamma(H) \leq 2\gamma(G\Box H)$.

In the talk we will discuss recent results related to Vizing's conjecture.

On vertices enforcing a hamiltonian cycle

Igor Fabrici

(joint work with Erhard Hexel and Stanislav Jendrol')

A nonempty set $X \subseteq V(G)$ of vertices of a hamiltonian graph G is called a *hamiltonian cycle enforcing set* (an *H-force set* in short) if every cycle C of G containing all vertices of X is hamiltonian. The smallest cardinality $h(G)$ of an H-force set in G is called the *H-force number* of G.

We present some results on the H-force number for k-connected graphs, planar graphs, prisms over graphs, and characterize graphs with small values of this invariant.

Decomposition of $K_{n,n}$ into $(0, j)$-prisms

Dalibor Frončekl

(joint work with Sylwia Cichacz)

We say that a graph H has a G-*decomposition* if there are subgraphs $G_0, G_1, \ldots, G_s$ of H, all isomorphic to G, such that each edge of H belongs to exactly one G_i. If the graph G (more precisely, each G_i, $i = 1, \ldots, s$) contains all vertices of H, then we say that H has a G-*factorization*.

In this talk we generalize the notion of prisms. Recall that a *prism* is a graph of the form $C_m \times K_2$. For j even let $(0, j)$-*prism* of order $4n$ be a graph with two cycles $C_{2n} = v_0, v_1, \ldots, v_{2n-1}$ and $C'_{2n} = v'_0, v'_1 \ldots, v'_{2n-1}$ and $2n$ additional edges $v_1v'_1, v_3v'_3, \ldots, v_{2n-1}v'_{2n-1}$ and $v_0v'_j, v_2v'_{2+j}, \ldots, v_{2n-j}v'_0$. In our terminology a prism is an $(0, 0)$-prism.

We will present in this talk some recent results on decompositions and factorizations of complete bipartite graphs into $(0, j)$-prisms.

Automata formalization for graphs of bounded rank-width

Robert Ganian

Rank-width is a brand new theoretical property of graphs, introduced by Oum and Seymour. The notion originated as a tool to better deal with graphs of bounded clique-width, yet rank-width is a completely self-contained, stand-alone property of graphs. It is natural to ask whether we can use the fact that a given class of graphs has bounded rank-width to design more efficient algorithms for this specific class of graphs.

There have been successful works which have shown that it is possible to use so-called parse trees to generate graphs of bounded tree-width and matroids of bounded branch-width, the general idea being that there exist many standardized tools for designing algorithms for parse trees. It should be noted that a graph has bounded rank-width if and only if it has bounded clique-width. However, the rank-width of a graph can be much smaller (even exponentially) than its clique-width, so designing algorithms through this new approach may lead to exponentially better results for a given class of graphs.

We first introduce our labeling formalism used for representing bipartite adjacency matrices of graphs, followed by a definition of parse trees for graphs of bounded rank-width. The final chapter of the work contains our proof of an analogue to the Myhill-Nerode theorem for our notion of parse trees for graphs of bounded rank-width.

Perfect parallel-edge-dominating coloring of the hypercube and Turán-type problems

Petr Gregor

Two edges of the n-dimensional hypercube Q_n are said to be *parallel* if they lie oppositely on a common 4-cycle. For $n = 2^r$ where r is an integer, we color properly the edges of Q_n with n colors (1-factorization) such that every edge has exactly one parallel edge of each color except its own color. Then we apply this coloring to some well-known Turán-type problems on the hypercube.

Hamiltonicity in $CN_{i,j,k}$-free graphs

Ruslan Gumerov

Denote by $N_{i,j,k}$ the graph which is obtained by identifying each vertex of a triangle with an endvertex of one of three vertex-disjoint paths of lengths i,j,k. There are following results on forbidden $C \simeq K_{1,3}$ and $N_{i,j,k}$ subgraphs and hamiltonicity.

Theorem (Duffus et al. 1980)**.** *Every 2-connected $CN_{1,1,1}$-free graph is hamiltonian.*

Theorem (Brousek et al. 1998)**.** *Every 3-connected $CN_{1,1,2}$-free and $CN_{1,2,2}N_{1,1,3}$-free graph is hamiltonian.*

We study an analogical problem for $CN_{i,j,k}$-free graphs, where $i+j+k \geqslant 5$.

Goal-minimally k-diametric graphs

Štefan Gyürki

A graph G with diameter k is said to be goal-minimally k-diametric if for every edge uv of G distance $d_{G-uv}(x, y) > k$ if and only if $\{x, y\} = \{u, v\}$. Kyš conjectures, that for every integer $k \geq 1$ there exists a k-GMD graph. As we will see, this conjecture has been proved for only few values of k. Moreover, there exists an infinite number of k-GMD graphs for $k = 1, 2, 4$ and 6.

On large light graphs in families of polyhedral graphs

Róbert Hajduk

(joint work with Roman Soták)

A graph H is said to be light in a family $\mathcal{H}$ of graphs if each graph $G \in \mathcal{H}$ containing a subgraph isomorphic to H contains also an isomorphic copy of H such that each its vertex has the degree (in G) bounded above by a finite number $\varphi(H, \mathcal{H})$ depending only on H and $\mathcal{H}$. We prove that in the family of all 3-connected plane graphs of minimum degree 5 (or minimum face size 5, respectively), the paths with certain small graphs attached to one of its ends are light.

Approaching tree-width of graphs from a matroidal perspective

Petr Hliněný

(joint work with Geoff Whittle)

The wide-spread notion of graph tree-width can nowadays be already considered "traditional". Its roots date back to beginnings of the Graph Minors project of Robertson and Seymour in the 80's, when several equivalent approaches to measuring tree-likeness of a graph emerged. (The major use of those being in algorithmic desing area.) Among the several equivalent traditional definitions of tree-width, all pay an explicit attention to graph vertices. However, while studying possible extensions of tree-width to matroids, Hliněný and Whittle in 2003 proposed a new, "vertex-free" equivalent definition of graph tree-width. We would like to give an overview of this new perspective, and to speak about some recent development in it.

Maximum weight of a graph of given order, size and property

Mirko Horňák

(joint work with Stanislav Jendrol' and Ingo Schiermeyer)

The weight of an edge $e = xy$ of a graph G is $w(e) := \deg_G(x) + \deg_G(y)$ and the weight of G is $w(G) := \min(w(e) : e \in E(G))$. For a positive integer n, $m \in \{0, \ldots, \binom{n}{2}\}$ and a graph property $\mathcal{P}$ let

$$w(n, m, \mathcal{P}) := \max(w(G) : |V(G)| = n, |E(G)| = m, G \in \mathcal{P}).$$

If $\mathcal{P}_1 \subseteq \mathcal{P}_2$, then $w(n, m, \mathcal{P}_1) \leq w(n, m, \mathcal{P}_2)$; furthermore, $w(n, m, \mathcal{P}_1) = w(n, m, \mathcal{P}_2)$ implies $w(n, m, \mathcal{P})$ is constant for any $\mathcal{P}$ with $\mathcal{P}_1 \subseteq \mathcal{P} \subseteq \mathcal{P}_2$.

At Czechoslovak Symposium on Combinatorics, Graphs and Complexity in 1990 Erdős posed the problem of determining $w(n, m, \mathcal{I})$ for the most general property $\mathcal{I}$ of all graphs. The problem has been solved first partially by Ivančo and Jendrol' in [1] and then completely by Jendrol' and Schiermeyer in [2].

Theorem. *Let* $\mathcal{P}_1 = \{G : G \text{ is connected}, |V(G)| \geq 2\}$, $\mathcal{P}_2 = \{G : \delta(G) \geq 1\}$ *and* $\mathcal{P}_1 \subseteq \mathcal{P} \subseteq \mathcal{P}_2$. *If* $n \geq 20$, $m \geq \frac{3n^2}{8}$ *and* $2m \equiv q \pmod{n}$ *with* $q \in \{0, \ldots, n-1\}$, *then*

$$w(n, m, \mathcal{P}) = \begin{cases} 2\lfloor \frac{2m}{n} \rfloor & for\ q < \lfloor \frac{2n}{m} \rfloor, \\ 2\lfloor \frac{2m}{n} \rfloor + 1 & for\ q \geq \lfloor \frac{2n}{m} \rfloor. \end{cases}$$

References

[1] J. Ivančo and S. Jendrol', On extremal problems concerning weights of edges of graphs, *Sets, graphs and numbers (Budapest, 1991)*, 399–410, Colloq. Math. Soc. János Bolyai, 60, North-Holland, Amsterdam, 1992.

[2] S. Jendrol' and I. Schiermeyer, On a max-min problem concerning weights of edges, *Combinatorica* 21 (2001), 351–359.

Hamilton cycles in star graphs

Tomáš Kaiser

(joint work with Roman Čada, Moshe Rosenfeld and Zdeněk Ryjáček)

The vertices of the *star graph* $\Sigma(n)$ are all the permutations of $\{1, \ldots, n\}$, and two permutations σ, τ are joined by an edge if τ can be obtained from σ by swapping two entries at positions 1 and $i \geq 2$. The Cayley graph $\Sigma(n)$ is known to be hamiltonian. We begin our talk with a review of known results on Hamilton cycles in $\Sigma(n)$ and related graphs, and sketch a proof of the recent result of Čada, Rosenfeld, Ryjáček and the speaker that $\Sigma(n)$ contains $\lfloor (n-1)/12 \rfloor$ edge-disjoint Hamilton cycles for any prime n. The problem has interesting connections to combinatorial Gray codes.

Face 2-colourable triangulations by $K_{n,n,n}$ and Latin squares

Martin Knor

Face 2-colourable triangulations by $K_{n,n,n}$ proved themselves to be very useful for obtaining lower bound for the number of nonisomorphic triangulations by complete graphs K_n. Recently, this lower bound is n^{cn^2} for some infinite class of values n and a constant c, and this bound is (up to the constant c) best possible. As the key role in the proof plays proving that there are at least n^{cn^2} nonisomorphic face 2-colourable triangulations by $K_{n,n,n}$, we discuss these embeddings. They correspond to biembeddings of (main classes of) Latin squares.

Spanning tree factorizations of complete graphs

Petr Kovář

(joint work with Dalibor Fronček, Tereza Kovářová and Michael Kubesa)

Graph factorizations have been studied for several decades. The focus of this talk is factorizations of complete graphs into isomorphic spanning trees.

After some historical notes we give a survey of methods used for spanning tree factorizations. These include mostly graph labelings, e.g. blended, swapping and fixing labelings and their extensions. We compare the spectrum of questions answered by each labeling or method.

Often graph factorizations are considered to be a special case of graph decompositions. The specifics of spanning tree factorizations introduce such limitations that make many common methods for graph decompositions unusable for this "special case". In the talk we try to outline the place of spanning tree factorizations among factorizations, decompositions, orthogonal double covers and packings.

In contrary to graph decompositions or regular graph factorizations the families of trees known to factorize complete graphs are rather sparse. Nonetheless some general answers including necessary conditions and some sufficient conditions are known and we give a survey of them.

In the last part of the talk open problems will be presented along with their expected difficulty.

Cycle covers of graphs

Daniel Kráľ

(joint work with Tomáš Kaiser, Bernard Lidický, Pavel Nejedlý and Robert Šámal)

Cycle covers of graphs form a prominent topic in graph theory closely related to several deep and open problems. A *cycle* in a graph is a subgraph with all degrees even. A *cycle cover* is a collection of cycles such that each edge is contained in at least one of the cycles (each edge is *covered*). The Cycle Double Cover Conjecture of Seymour and Szekeres asserts that every bridgeless graph G has a collection of cycles containing each edge of G exactly twice (*cycle double cover*). Cycle Double Cover Conjecture is known to be implied by several other conjectures, among others the Shortest Cycle Cover Conjecture of Alon and Tarsi which asserts that every bridgeless graph with m edges has a cycle cover of total length at most $7m/5$.

In this talk, we will review known results on cycle covers of graphs and report on recent results on the Shortest Cycle Cover Conjecture obtained by the speaker and his coauthors. The best known general result on short cycle covers is due to Alon and Tarsi (1985) and Bermond, Jackson and Jaeger (1983): every bridgeless graph with m edges has a cycle cover of total length at most $5m/3 \approx 1.667m$. The general bound was improved for the class of cubic bridgeless graphs by Fan (1994) to $44m/27 \approx 1.630m$. We strengthen Fan's result in two directions: we establish the bound of $34m/21$ for the class of cubic graphs and extend Fan's bound of $44m/27$ to the class of graphs with minimum degree three. The known improvements of the original bound of $5m/3$ may seem to be rather minor, however, obtaining a bound below $8m/5 = 1.600m$ could be quite tricky since this bound is implied by Tutte's 5-Flow Conjecture.

Generalized domination in degenerate graphs

Jan Kratochvíl

(joint work with Petr Golovach)

The so called (σ, ρ)-domination, introduced by J. A. Telle, is a concept which provides a unifying generalization for many variants of domination in graphs. (A set S of vertices of a graph G is called (σ, ρ)-*dominating* if for every vertex $v \in S$, $|S \cap N(v)| \in \sigma$, and for every $v \notin S$, $|S \cap N(v)| \in \rho$, where σ and ρ are sets of nonnegative integers and $N(v)$ denotes the open neighborhood of the vertex v in G.) It is known that for any two nonempty finite sets σ and ρ (such that $0 \notin \rho$), the decision problem whether an input graph contains a (σ, ρ)-dominating set is NP-complete, but that when restricted to some graph classes, polynomial time solvable instances occur. We show that for every k, the problem performs a complete dichotomy when restricted to k-degenerate graphs, and we fully characterize the polynomial and NP-complete instances. It is further shown that the problem is polynomial time solvable if σ, ρ are such that every k-degenerate graph contains at most one (σ, ρ)-dominating set, and NP-complete otherwise. This relates to the concept of ambivalent graphs previously introduced for chordal graphs.

On the $(k, n-k)$-hypomorphic graphs

Naďa Krivoňáková

(joint work with Peter Czimmermann)

The following proposition generalizes the famous Reconstruction Conjecture: if two graphs are hypomorphic, they have to be also isomorphic.

In this contribution we discuss certain possibilities to generalize the term hypomorphism, also called the $(k, n-k)$-hypomorphism. We study properties of $(k, n-k)$-hypomorphic graphs and possible relationship of $(k, n-k)$-hypomorphism with some else well-known generalizations of the term hypomorphism.

Snarks with given real flow number

Robert Lukot'ka

(joint work with Martin Škoviera)

Pan and Zhu [*Construction of graphs with given circular flow numbers*, J. Graph Theory 43 (2003), 304–318] construct infinitely many graphs with real flow number r for every rational number r, $4 < r < 5$. Moreover these graphs are 4-cyclically edge connected, so they can be used to construct snarks by replacing every vertex of degree higher than four with a cubic network. We show how to make this construction in such a way that we preserve both the cyclic connectivity and the real flow number. This construction gives infinitely many snarks with real flow number r for every rational number r, $4 < r < 5$.

Cycles and matchings in cubic graphs

Edita Máčajová

The Cycle Double Cover Conjecture claims that every bridgeless graph contains a family of cycles that together cover each edge exactly twice. This conjecture is equivalent to its restriction to the family of cubic graphs. The Fulkerson Conjecture asserts that any bridgeless cubic graphs contains a family of six perfect matchings that together cover every edge exactly twice.

We discuss old and new results and research directions on the way towards these long-standing conjectures. Our talk will include results concerning extensions, bipartizing matchings, Fano colourings, and others. In particular, we sketch a result of a joint work with Martin Škoviera, that every bridgeless cubic graph of oddness 2 admits a Fano F_4-colouring – making a step towards the Fulkerson Conjecture.

Theory of light graphs and related topics

Tomáš Madaras

A graph H is defined to be *light* in a family $\mathcal{H}$ of graphs if there exists a finite number $w(H, \mathcal{H})$ such that each $G \in \mathcal{H}$ which contains H as a subgraph, contains also a subgraph $K \cong H$ such that the sum of degrees (in G) of the vertices of K (that is, the weight of K in G) is at most $w(H, \mathcal{H})$. Such a definition allows to to compare, unify and generalize previously known results on local structure of graphs of various graph families (particularly, of plane graphs). We give an overview of the development and results of light graphs theory, and present some related concepts based on generalizations and analogues of the notion of light graph.

Circular chromatic index of Blanuša snarks

Ján Mazák

Recently, a graph invariant called the circular chromatic index has attracted much attention. For $r > 0$, a *circular r-edge-colouring* of a graph G is a mapping $c\colon E(G) \to [0, r)$ such that $1 \leq |c(e) - c(f)| \leq r - 1$ for any two adjacent edges e and f of G. The *circular chromatic index* of G, denoted by $\chi'_c(G)$, is the infimum of all $r > 0$ such that G has a circular r-edge-colouring. This infimum is in fact a minimum – for a finite graph it is always attained. Moreover, it is rational. It is not difficult to show that $\chi'(G) = \lceil \chi'_c(G) \rceil$, so the circular chromatic index is a refinement of the usual chromatic index.

The circular chromatic index is most interesting for bridgeless cubic graphs that are not 3-edge-colourable – snarks. Any snark has its circular chromatic index greater than 3. From the results of Kochol (1996) and Kaiser, Král' and Škrekovski (2004) it follows that there exist an infinite class of snarks with the index converging to 3, but no such class has been known. In fact, there have been only a finite number of values of the index of snarks known, namely the values for the Isaac's snarks, Goldberg snarks and for the Petersen graph. In our work we determine the exact value of the circular chromatic index of generalized Blanuša snarks of type 1 introduced by Watkins more than two decades ago. In this case, the index takes infinitely many values and can get arbitrarily close to 3.

Branch-decomposition applied to route-planning

Ondrej Moriš

In this talk we present the real-world application of a branch-decomposition. We are concerned with route-planning in huge graphs representing real-world road maps: for a given pair of vertices of a road map we want to very effectively find optimal path between them. The application area of this are car GPS navigation systems. We briefly introduce basic principles of our approach based on a branch-decomposition and a bidirectional search. We also present some examples of our experimental results on real road maps of Europe. Finally, some of possible improvements and trends of future work are indicated.

On renormalization of scale-free networks

Martin Nehéz

(joint work with Pavel Paroulek)

Scale-free networks are characterized by a power-law degree distribution of nodes. This model is a kind of complex networks and it is used for modeling of many real-world networks, such as social, engineering and computer networks.

From a mathematical perspective, the renormalization is a collection of techniques used to take a continuum limit. More specifically, it allows to use a conversion from countable discrete structures to the continuum. Its applications are known mostly in physics, such as quantum field theory and statistical mechanics. From a graph-theoretical point of view, the renormalization can be understand as a graph transformation which preserves the scale-free property as an invariant. Finally, from an algorithmic point of view, the renormalization of scale-free networks is a special type of a graph partitioning problem which is in NP. Our formalization of this problem allows us to study its general properties. We design a heuristics which solves the problem of iterated renormalization of scale-free networks. Our heuristics DBBC (degree based box covering) is based on the degree-preference strategy. We study its properties both by analytical methods and also by experimental evaluation.

Local algorithms for Unit Disk graphs

Jaroslav Opatrny

(joint work with Evangelos Kranakis, Ladislav Stacho et al.)

Many graph theoretical problems remain NP-complete for Unit Disk graphs and existing distributed algorithms for approximations of the problems are not local. Using information on the location of nodes we give local algorithm for the dominating set and connected dominating set problem with constant approximation factor. We show that other problems can be similarly solved.

Recognition problem and sandwiching

Martin Pergel

(joint work with Jan Kratochvíl)

There are many classes of intersection graphs studied to obtain efficient algorithms for generally hard problems. The *recognition problem* (to decide whether a given graph has an appropriate intersection representation) is a very important problem, as many efficient algorithms require an appropriate intersection representation. When a class appears to be too hard to recognize, it is natural to ask if there is some other “relatively similar” class easy to be recognized. String graphs are intersection graphs of arc-connected curves in the plane, segment, (resp. pseudosegment) graphs are intersection graphs of straight line segments (or pseudosegments, resp.) in the plane, pseudodisk graphs and graphs of homothetic triangles are similarly intersection graphs of respective objects in the plane. The *sandwiching problem* refers to a question, given classes $\mathcal{S}$ and $\mathcal{T}$ such that $\mathcal{S} \subseteq \mathcal{T}$ whether there exists a polynomially recognizable class $\mathcal{U}$ such that $\mathcal{S} \subseteq \mathcal{U} \subseteq \mathcal{T}$.

In 1990 Kratochvíl showed that between segment- and string-graphs no polynomially recognizable class can be sandwiched. We present some other reductions. Namely we show non-existence of polynomially recognizable class sandwiched between class of polygon-circle graphs (intersection graphs of convex polygons inscribed into a circle) and graphs of interval filaments (curves above a prescribed line with both endpoints on the line; only curves over non-disjoint intervals are allowed to intersect each other). As well we show that no polynomially recognizable class lies between class of pseudodisk graphs and graphs of homothetic polygons in the plane.

Strong parity vertex coloring of outerplanar graphs

Ondřej Rucký

(joint work with Tomáš Kaiser)

Let c be a coloring of a 2-connected plane graph G. In [1], the following notion is introduced: c is the *strong parity vertex coloring* (spv coloring shortly) of G if for each face F of G and each color c_i of c, no vertex or an odd number of vertices incident with F are colored by c_i. The minimum of sizes of all spv colorings of G is denoted by $\chi_s(G)$.

It is easy to find graphs with $\chi_s(G) = 6$, though no graph with $\chi_s(G) > 6$ is known at present. Hence, the authors of [1] conjecture that there exists an integer k such that $\chi_s(G) \leq k$ for all plane graphs, and they believe that $k = 6$.

We deal with this conjecture for the class of outerplanar embeddings of graphs. We prove that indeed $\chi_s(G) \leq 6$ for any G of this class, and that $\chi_s(G) \leq 4$ provided G is bipartite.

References

[1] J. Czap and S. Jendrol', Colouring vertices of plane graphs under restrictions given by faces, *Discussiones Math. Graph Theory* (submitted).

On super (a, d)-edge-antimagic total labelings of disconnected graphs

Andrea Semaničová

(joint work with Martin Bača and Yuqing Lin)

A *labeling* of a graph is a mapping that carries some sets of graph elements into numbers (usually the positive integers). An (a, d)-*edge-antimagic total labeling* of a graph $G(V, E)$ is a one-to-one mapping f from $V(G) \cup E(G)$ onto the set $\{1, 2, \ldots, |V(G)| + |E(G)|\}$, such that the set of all the edge-weights, $w_f(uv) = f(u) + f(uv) + f(v), uv \in E(G)$, forms an arithmetic sequence starting from a and having a common difference d. Such a labeling is called *super* if the smallest possible labels appear on the vertices.

In this talk we will deal with the super (a, d)-edge-antimagic total labelings of disconnected graphs for $d = 0, 1, 2, 3$.

Rainbow colourings

Ingo Schiermeyer

(joint work with Ralph J. Faudree and Hao Li)

An edge-coloured graph is called *rainbow* if no two edges have the same colour. Given a host graph G and a subgraph H of G, let $f(G,H)$ denote the maximum number of colours in any edge-colouring of G with no rainbow copy of H. Equivalently, any edge-colouring of G with at least $rb(G,H) = f(G,H)+1$ colours contains a rainbow copy of H. The numbers $rb(G,H)$ will be called rainbow numbers.

In this talk we will give a survey about rainbow numbers for various classes of graphs.

On the LOGO and the exponentiation of snarks

Zdzisław Skupień

(joint work with Ela Gebus)

Conjecture. *The genuine snarks do not exist.*

Theorem. *There are at least exponentially many n-vertex hypohamiltonian snarks with cyclic edge-connectivity five and girth either 5 or 6 as well as that many with cyclic edge-connectivity six.*

Partially t-balanced Cayley maps

L'ubica Staneková

A Cayley map is an embedding of a Cayley graph in which the cyclic ordering of generators around every vertex is the same. The involution that indicates the position of mutually inverse generators in the cyclic ordering is called the distribution of inverses of a Cayley map. The Cayley maps whose distribution of inverses is given by a linear function modulo the valency of the map are called t-balanced.

In our contribution we relax the defining condition of t-balancedness, resulting in Cayley maps which we call partially t-balanced, and include a discussion on the origin, motivation, and results related to the new concept.

Some parameterized problems related to Seidel's switching

Ondřej Suchý

Seidel's switching of vertex set is an operation, which deletes edges leaving this set from the graph and adds those edges between the set and the rest of the graph, that weren't there originally. Other edges remain untouched by this operation. The usual question in parameterized complexity is whether the exponential part of the algorithms for hard problems can be bounded by some function of only selected parameter, which we assume to be small. We study the complexity of a question, if the given graph can be turned into a graph with some property P using Seidel's switching, from the parameterized view. We show fixed-parameter tractability of switching to a regular graph, to a graph with bounded degree of vertices, or with bounded number of edges, a graph without a forbidden subgraph and a bipartite graph.

Spectra of (3,6)-fullerenes

Robert Šámal

(joint work with Matt DeVos, Luis Goddyn and Bojan Mohar)

We determine the spectra of cubic plane graphs whose faces have sizes 3 and 6. Such graphs, "(3,6)-fullerenes", have been studied by chemists who are interested in their energy spectra. In particular we prove a conjecture of Fowler, which asserts that all their eigenvalues come in pairs of the form $\{\lambda, -\lambda\}$ except for the four eigenvalues $\{3, -1, -1, -1\}$. We exhibit other families of graphs which are "spectrally nearly bipartite" in this sense. Our proof utilizes a geometric representation to recognize the algebraic structure of these graphs, which turn out to be examples of Cayley sum graphs.

Structure of noninscribability

Sergej Ševec

The phenomenon of polyhedra noninscribability is explored. New sufficient conditions are presented in historical seqence and context of the well-known results of Steinitz, Grunbaum, and Jucovic. In addition to these graph-combinatorial properties assuring noninscribability of large classes of polyhedral graphs, some special new constructions and procedures are presented that enlarge the variety of noninscribable types even further. The focus is on combinatorial descriptions, relationship with other approaches is discussed and basic proof techniques and principle arguments are enlightened.

Covering projection of regular graphs

Marek Tesař

A covering projection from graph G onto graph H is "local isomorphism": a mapping from the vertex set of G onto the vertex set of H such that, for every $v \in V(G)$, the neighborhood of v is mapped bijectively onto the neighborhood (in H) of the image of v. We study the computational complexity of the H-cover (deciding if a given graph G covers H), where G is a regular graph with 8 vertices and edges of two colors, where edges of one color create two disjoint 4-cycles. We present full characterization of H-cover problem for such 3-regular graphs. We solve polynomial cases by reduction to system of linear equations and we show some graphs for which this method doesn't work (even though H-cover is polynomial).

Antibandwidth of d-dimensional meshes

L'ubomír Török

(joint work with Imrich Vrt'o)

The antibandwidth problem is to label vertices of a graph $G = (V, E)$ bijectively by $0, 1, 2, , ..., |V| - 1$ such that the minimal difference of labels of adjacent vertices is maximised. We present the labelling algorithm and prove its asymptotical optimality for antibandwidth value of d-dimensional meshes $P_n \times P_n \times ... \times P_n$.

On embedding triconnected cubic graphs on point sets

Pavel Valtr

(joint work with
Alfredo García, Ferran Hurtado, Clemens Huemer and Javier Tejel)

Let S be a set of n points in general position in the plane, let $\text{ext}(S)$ be the number of extremal points of S, and let $\text{ext}(S) < n$. Then we construct a 3-connected plane graph on S with $\max\{\lceil 3n/2 \rceil, n + \text{ext}(S) - 1\}$ edges, and we show that there is no 3-connected plane graph on S with a smaller number of edges. In particular, this implies that S admits a 3-connected cubic plane graph if and only if $\text{ext}(S) \leq n/2 + 1$. We also give a partitial characterization of planar point sets admitting a cubic plane graph which is not necessarily 3-connected.

Distance regular square of distance regular graphs

Vladimír Vetchý

Let X be a finite set, $\operatorname{card} X \geq 2$. For an arbitrary natural number D let $R = \{R_0, R_1, \ldots, R_D\}$ be a system of binary relations on X. Let a pair (X,R) be *an association scheme* with D classes.

Given an undirected graph $G = (X, E)$ of diameter D we may define $R_k = \{(x, y); d(x, y) = k\}$, where $d(x, y)$ is the distance from the vertex x to the vertex y in the standard graph metric. If (X, R) gives rise to an association scheme, the graph G is called *distance regular* (for $D = 2$ is called *strongly regular*).

The necessary conditions for distance regular graph G to have the square G^2 strongly regular are found and the same it is solved for some distance regular bipartite graphs.

Graph coverings in constructions of large graphs and digraphs with given diameter and degree

Mária Ždímalová

Interconnection networks are often modelled by graphs and digraphs. The most common types of restrictions considered in such models are upper bounds on vertex degrees and on the diameter. The task then is to construct largest possible graphs and digraphs with bounded degree and diameter, which is known as the *degree-diameter problem* in the literature. It is often desirable to require extra properties of such large graphs, and a frequently considered property in this context is vertex-transitivity.

Along with a number of other construction methods, the covering graph technique has recently been successfully used to construct large graphs of given degree and diameter. In our contribution we will show that some of the families of large vertex-transitive digraphs discovered earlier can also be described in terms of graph coverings.

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