Preface

This is a collection of abstracts of some of the talks at the Hamiltonian Graph Theory Workshop, held in Pavlov (Czech Republic) on February 6–9, 2008, to honor the sixtieth birthday of Professor Zdeněk Ryjáček. We hope that all the participants — especially Zdeněk — enjoyed these four days of interesting talks in the friendly ambience of Hotel Pavlov as much as we did.

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We thank all the participants for accepting the invitation to come and contribute to the atmosphere of this event.

Roman Čada, Přemysl Holub, Tomáš Kaiser,
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Stable properties and around
Z. Ryjáček

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Abstracts of some talks

On labelings of disconnected graphs

Martin Bača

TU Košice
martin.baca@tuke.sk

A *labeling* of a graph is any map that carries some set of graph elements to numbers (usually to the positive integers). If the domain is the vertex-set or the edge-set, the labelings are called *vertex labelings* or *edge labelings*, respectively. Moreover, if the domain is $V(G) \cup E(G)$ then the labeling is called *total labeling*.

Let f be a vertex labeling of a graph G, we define the *edge-weight* of $uv \in E(G)$ to be $w(uv) = f(u) + f(v)$. If f is a total labeling, then the edge-weight of uv is $w(uv) = f(u) + f(uv) + f(v)$.

An (a, d)-*edge-antimagic total labeling* on a graph with p vertices and q edges is defined as a one-to-one map taking the vertices and edges onto the integers $1, 2, \ldots, p + q$ with the property that the edge-weights form an arithmetic sequence starting from a and having a common difference d. Such a labeling is called *super* if the smallest possible labels appear on the vertices.

A *graceful labeling* of a (p, q) graph G is an injection $h : V(G) \to \{1, 2, \ldots, q + 1\}$ such that, when each edge uv is assigned the label $|h(u) - h(v)|$, the resulting edge labels are distinct. When the graceful labeling h has the property that there exists an integer λ such that for each edge uv either $h(u) \leq \lambda < h(v)$ or $h(v) \leq \lambda < h(u)$, h is called an α-*labeling*.

We use the connection between α-labelings and edge-antimagic labelings for determining a super (a, d)-edge-antimagic total labelings of disconnected graphs.

Colouring and distinguishing edges by total labellings

Stephan Brandt

TU Ilmenau
stephan.brandt@tu-ilmenau.de

(joint work with Kristína Budajová, Jozef Miškuf, Dieter Rautenbach and Michael Stiebitz)

A total k-labelling of a graph $G = (V, E)$ is a function $f : V \cup E \to \{1, 2, \dots, k\}$. The weight of an edge uv is $w(uv) = f(u) + f(uv) + f(v)$. We investigate edge-distinguishing total k-labellings, where all edge weights must be different, and edge-colouring total k-labellings, where the edge weights of incident edges must be different, i.e. they determine a proper edge colouring of G. In both cases we try to minimize k.

Let G be a graph with m edges and maximum degree Δ. In the case of edge-distinguishing total labellings, our main result is that the natural lower bound

$$k \geq \left\lceil \max\left\{\frac{m+2}{3}, \frac{\Delta+1}{2}\right\}\right\rceil$$

is tight for all graphs with $m \geq 111000\Delta$. Ivančo and Jendrol' conjecture that the bound is tight for all $G \neq K_5$.

In the case of edge-colouring total labellings the natural lower bound is $k \geq \lceil \frac{\Delta+1}{2} \rceil$. This lower bound cannot be tight in general, but we are not aware of any graph, where k must exceed the lower bound by more than one. Our main result here is an upper bound of $k \leq \frac{\Delta}{2} + \mathcal{O}(\sqrt{\Delta \log \Delta})$. In both cases we employ a mixture of graph theoretic and probabilistic methods.

Sharp upper bounds for the minimum number of components of 2-factors in claw-free graphs

Hajo Broersma

Durham University
hajo.broersma@durham.ac.uk

(joint work with Daniël Paulusma and Kiyoshi Yoshimoto)

We first note that for claw-free graphs on n vertices with minimum degree $\delta = 2$ or $\delta = 3$ that have a 2-factor we can not do better than the trivial upper bound $n/3$ on the number of components of a 2-factor. Hence, in order to get a nontrivial result it is natural to consider claw-free graphs with $\delta \geq 4$. Let G be a non-hamiltonian claw-free graph on n vertices with minimum degree δ. We prove the following results, thereby improving known results due to Faudree et al. and to Gould & Jacobson. If $\delta = 4$, then G has a 2-factor with at most $(5n-14)/18$ components, unless G belongs to a finite class of exceptional graphs. If $\delta \geq 5$, then G has a 2-factor with at most $(n-3)/(\delta-1)$ components. These bounds are best possible in the sense that we cannot replace $5/18$ by a smaller quotient and we cannot replace $\delta - 1$ by δ.

Forbidden subgraphs, Hamiltonian properties, and 2-factors in graphs

Ralph Faudree

University of Memphis
rfaudree@cc.memphis.edu

A survey of results on families of connected forbidden subgraphs that imply various hamiltonian type properties will be presented. Some corresponding results on the existence of two-factors will also be presented. Applications of the Ryjáček closure to results of this type will be featured. Open problems will be discussed.

Application of linear algebra for the existence of homomorphisms with local constraints

Jiří Fiala

Univerzita Karlova, Praha
fiala@kam.mff.cuni.cz

(joint work with Daniël Paulusma and Jan Arne Telle)

We explore the connection between locally constrained graph homomorphisms and degree matrices arising from an equitable partition of a graph. We extend the well-known connection between degree refinement matrices of graphs and locally bijective graph homomorphisms to locally injective and locally surjective homomorphisms by showing that also these latter types of homomorphisms impose a quasiorder on degree matrices and a partial order on degree refinement matrices. Computing the degree refinement matrix of a graph is easy, and an algorithm deciding comparability of two matrices in one of these partial orders could be used as a heuristic for deciding whether a graph G allows a homomorphism of the given type to H. By using elementary properties of systems of linear equations we show for local surjectivity and injectivity that the problem of matrix comparability belongs to the complexity class NP.

On a problem from the ancient times of Zdenek's youth

Dalibor Frončekk

University of Minnesota Duluth
dalibor@d.umn.edu

Let G be a simple finite graph and x, y be two independent (i.e., non-adjacent) vertices. By $N^i_G(x, y)$ we denote the subgraph of G induced by the set of all vertices adjacent to at least one of x, y.

G is a *graph with constant neighborhood of two independent vertices* if there exists a graph H such that $N^i_G(x, y)$ is isomorphic to H for every pair of independent vertices x, y. It is known (see [1]) that if G has this property, then $\mathrm{diam}(G) \leq 3$. On the other hand, no such graph with $\mathrm{diam}(G) = 3$ is known. Therefore, we may ask the following question.

Problem. *Does there exist a graph with constant neighborhood of two independent vertices of diameter 3?*

It is easy to show the following:

Observation. *Let G be a graph with constant neighborhood of two independent vertices and U be the set of all vertices with eccentricity 3. Then $|U| \geq 3$. Also, G cannot contain an induced cycle of length 6 or more.*

References

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Seidel's switching

Jan Kratochvíl

Univerzita Karlova, Praha
honza@kam.mff.cuni.cz

Let G be a graph. *Seidel's switching* of a vertex $v \in V_G$ results in a graph called $S(G, v)$ whose vertex set is the same as of G and the edge set is the symmetric difference of E_G and the full star centered in v, i.e.,

$$V_{S(G,v)} = V_G$$

$$E_{S(G,v)} = (E_G \setminus \{xv : x \in V_G\}) \cup \{xv : x \in V_G, x \neq v, xv \notin E_G\}.$$

Graphs G and H are called *switching equivalent* if G can be transformed into a graph isomorphic to H by a sequence of Seidel's switches. It can be easily seen that only the parity of the number of times a particular vertex is switched matters. Denote $A \subseteq V_G$ the set of vertices which are switched odd number of times. The resulting switched graph is then

$$S(G, A) = (V_G, E_G \div \{xy : x \in A, y \in V_G \setminus A\}),$$

and G is switching equivalent to H if and only if H is isomorphic to $S(G, A)$ for some $A \subseteq V_G$ ($\div$ denoting the symmetric difference of sets).

The concept of Seidel's switching was introduced by the Dutch mathematician J. J. Seidel in connection with symmetric structures, often of algebraic flavor, such as systems of equiangular lines, strongly regular graphs, or the so called two-graphs. For more structural properties of two-graphs, cf. [6, 7, 8].

Colbourn and Corneil [1] (and independently but later Kratochvíl et al. [4]) proved that deciding if two graphs are switching equivalent is an isomorphism complete problem. Several authors asked the question of how difficult it is to decide if a given graph is switching equivalent to a graph having some prescribed property (this property becomes the parameter of the problem). So far the only nontrivial switching NP-complete problem known is switching to a regular graph [4, 5].

An area with irritating open problems is avoiding forbidden induced subgraphs. It is still an open problem if there exists a graph H such that switching to H-free graphs is NP-complete. Here H is a fixed parameter. The complexity is known only for a few graphs H and in all cases the question turns out polynomially solvable. It is proved in [4] that deciding if a given input graph can be switched to a P_3-free graph (i.e., a graph not containing and induced copy of the path on 3 vertices) is polynomially solvable. This means deciding if the input graph is switching equivalent to the disjoint union of complete graphs. R. Hayward [2] showed that deciding switching equivalence to triangle-free graphs is also polynomial.

Another case is added in [3]. This result is particularly suitable to be presented in the Ryjáček volume, and we wish to add the following theorem to the pile of birthday presents:

Theorem. *It is polynomial to decide if an input graph is switching equivalent to a* **claw-free** *graph.*

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Some Ramsey type problems surrounding claw-free graphs

Jaroslav Nešetřil

Univerzita Karlova, Praha
nesetril@kam.mff.cuni.cz

Let G, H be finite graphs. We say that H is *vertex-* (or *edge-*) *Ramsey* for G if for every partition $\mathcal{A}_1 \cup \mathcal{A}_2$ of vertices $V(H)$ (or edges $E(H)$) there exists an inducd subgraph G' of H such that $V(G') \subset \mathcal{A}_i$ (or $E(G') \subset \mathcal{A}_i$) for either $i = 1$ or $i = 2$.

In the style of Erdős-Rado partiton arrow [1, 2] this is denoted by $H \longrightarrow (G)^1_2$ (or $H \longrightarrow (G)^2_2$). One more definition (a key one which took a long time to crystalize, see e.g. [2]): Let $\mathcal{K}$ be a class of finite graphs. We say that $\mathcal{K}$ is *vertex-Ramsey* (or *edge-Ramsey*) if for every $G \in \mathcal{K}$ there exists $H \in \mathcal{K}$ such that $H \longrightarrow (G)^1_2$ (or $H \longrightarrow (G)^2_2$).

Known results for Ramsey classes deal with "rich" classes of graphs. For example we have

Theorem. *The class GRA of all fnite graphs is both vertex- and edge-Ramsey.*

Theorem. *For any $k \geq 2$ is the class FORB(K_k) of all K_k-free graphs both vertex- and edge-Ramsey.*

Theorem. *The class BIP of all bipartite graphs is edge-Ramsey (and obviously not vertex-Ramsey).*

(For these classical results, see, e.g., the survey [2].)

By specializing to more structures classes of graphs we get several interesting results and problems. For example we have:

Theorem. *The class INT of all interval graphs s vertex-Ramsey but not edge-Ramsey.*

The class UNIINT of all unit interval graphs is both vertex-Ramsey and edge-Ramsey.

Theorem. *The class PERFECT of all perfect graphs is vertex-Ramsey.*

It is my old problem to decide whether perfect graphs are edge-Ramsey class. This is an open problem even after the solution of Perfect Graph conjecture (by Chudnovsky, Robertson, Seymour and Thomas).

The classes of bounded degree graphs (and cubic graphs in particular) fail to be Ramsey. However the class CLAW of all claw free graphs (prominently treated in this volume) is interesting in this context:

Both principal building clases of the class CLAW, namely the class LINE of of all line graphs and the class 2IND of all graphs with their independence number ≤ 2, are vertex-Ramsey. One should note that while the class LINE fails to be edge-Ramsey, the class 2IND is both vertex- and edge-Ramsey. However it follows from the Chudnovsky–Seymour characterization of the class CLAW that this class fails to be even vertex-Ramsey. Are there only finitely many such examples? More exactly: Is it true that, with finitely many exceptions, for every claw free graph G there exists a claw free graph H such that $H \longrightarrow (G)^1_2$?

Let us finish this extended abstract (the proofs of which will of course appear elsewhere) by mentioning the class TRIANG of triangulated (or rigid circuit graphs). Here the situation is surprisingly open: The class TRIANG is known to be neither vertex- nor edge-Ramsey. The difficulty of treating this example may be related to the above problem for the class PERFECT.

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Stable properties and around

Zdeněk Ryjáček

Západočeská univerzita, Plzeň
ryjacek@kma.zcu.cz

For a claw-free graph G, $\mathrm{cl}(G)$ denotes its closure (obtained by local completions at locally connected vertices). A class of graphs $\mathcal{C}$ is said to be *stable* under the closure if $G \in \mathcal{C} \Rightarrow \mathrm{cl}(G) \in \mathcal{C}$. A property $\mathcal{P}$ is said to be *stable* in a stable class $\mathcal{C}$ if, for any $G \in \mathcal{C}$, G has $\mathcal{P} \Leftrightarrow \mathrm{cl}(G)$ has $\mathcal{P}$. A graph invariant π is said to be *stable* in a stable class $\mathcal{C}$ if, for any $G \in \mathcal{C}$, $\pi(G) = \pi(\mathrm{cl}(G))$. In the talk we survey known results on stability of graph properties under $\mathrm{cl}(G)$ and show some applications of closure techniques using the concept of stability. We also show some variations of the closure concept and the respective stability results. As a recent application it is shown (joint work with Petr Vrána) that every 7-connected claw-free graph is Hamilton-connected.

The following table summarizes known results on stability of graph properties under $\mathrm{cl}(G)$.

Property / invariant	Stable	Connectivity
Circumference	YES	1
Hamiltonicity	YES	1
Having a 2-factor with $\leq k$ components	YES	1
Minimum number of components in a 2-factor	YES	1
Having a cycle cover with $\leq k$ cycles	YES	1
Minimum number of cycles in a cycle cover	YES	1
(Vertex) pancyclicity	NO	any $\kappa \geq 2$
(Full) cycle extendability	NO	any $\kappa \geq 2$
Length of a longest path	YES	1
Traceability	YES	1
Having a path factor with $\leq k$ components	YES	1 [Ishizuka]
Min. number of components in a path factor	YES	1
Having a path cover with $\leq k$ paths	YES	1 [Ishizuka]
Minimum number of paths in a path cover	YES	1
	NO	3
Homogeneous traceability	???	$4 \leq \kappa \leq 6$
	YES	7
	NO	3
Hamilton-connectedness	???	$4 \leq \kappa \leq 6$
	YES	7 [Z.R., Vrána]

Having a P_3-factor	NO ??? YES	1 2 3 [Kaneko]
Flower property	YES	1
Hamiltonian index	YES	1
Having hamiltonian prism	YES	1 [Čada]

Closures, cycles and paths

Ingo Schiermeyer

TU Freiberg
schierme@math.tu-freiberg.de

(joint work with Arnfried Kemnitz, Jochen Harant and Akira Saito)

In 1960 Ore proved the following theorem: Let G be a graph of order n. If $d(u) + d(v) \geq n$ for every pair of nonadjacent vertices u and v, then G is hamiltonian. Since then for several other graph properties similar sufficient degree conditions have been obtained, so called "Ore-type degree conditions". In 2000, Faudree, Saito, Schelp and Schiermeyer strengthened Ore's theorem as follows: They determined the maximum number of pairs of nonadjacent vertices that can have degree sum less than n (i.e. violate Ore's condition) but still imply that the graph is hamiltonian. In this talk we will show that for some other graph properties the corresponding Ore-type degree conditions can be strengthened as well. These graph properties include traceable graphs, hamiltonian connected graphs, k-leaf connected graphs, pancyclic graphs and graphs having a 2-factor with two components. Graph closures are computed to show these results.

Distance regular square of distance regular graph

Vladimír Vetchý

Univerzita obrany, Brno
vladimir.vetchy@unob.cz

Given an undirected graph $G = (X, E)$ of diameter D we define $R_k = \{(x, y); d(x, y) = k\}$, where $d(x, y)$ is the distance from the vertex x to the vertex y in the standard graph metric. If (X,$\mathbf{R}$) gives rise to an association scheme, the graph G is called *distance regular*.

Let $G = (X, E)$ be an undirected graph without loops and multiple edges. *The second power* (or *square* of G) is the graph $G^2 = (X, E')$ with the same vertex set X and in which mutually different vertices are adjacent if and only if there is at least one path of the length 1 or 2 in G between them.

The necessary conditions for G to have the square G^2 distance regular are found and some constructions of those graphs are solved for distance regular graphs of diameter $D = 3$ and for distance regular bigraphs of diameter $D =$ 3, 4, 5, 6 and 7.