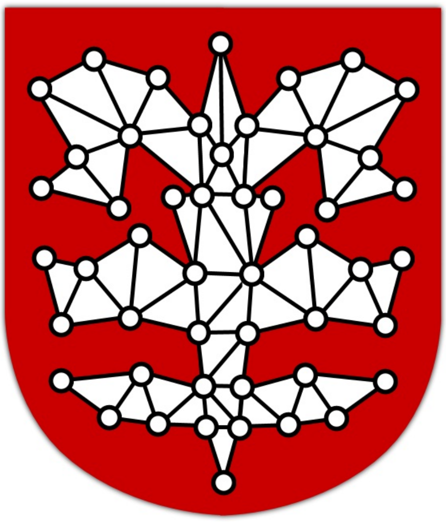
ČS Grafy 2012

47th Czech-Slovak conference on graph theory and combinatorics



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Preface

Let me greet you in Litomyšl, a picturesque town on the border between Bohemia and Moravia, a town famous for its historical and cultural monuments for which it has been listed among UNESCO cultural heritage sites.

In our country it is well known that Litomyšl is the birthplace of Bedřich Smetana, also that Alois Jirásek spent fourteen years on the local high school, and that several other well known cultural characters like Božena Němcová, Julius Mařák or Josef Váchal spent their lives here.

It is less known that Prof. Karel Zahradník was born here on April 16, 1848. He was a Czech mathematician, who spent a major part of his lifetime in Croatia, but later in 1899 he became the first rector of the University of Technology in Brno. Already before his leave for Croatia he had participated on the foundation of the Union of Czech Mathematicians and Physicists, a scholarly society which celebrates the 150th anniversary of its foundation this year.

Our conference on graph theory does not carry such long tradition like the Union of Czech Mathematicians and Physicists. Since the first conference which has been in 1961 in Liblice, we meet for the 47th time. Among others, Prof. Miroslav Fiedler gave birth of this tradition and I am very happy that I might welcome him here this year.

I believe that this year's conference will succeed to fulfill your expectations. Our invited talks shall essentially contribute to this intent. These will be given by

- Hajo Broersma, (University of Twente, The Netherlands),
- Katarína Cechlárová (P. J. Šafárik University, Košice, Slovakia),
- Tomáš Kaiser (West Bohemia University, Pilsen, Czech Republic),
- Jan Obdržálek (Masaryk University, Brno, Czech Republic),
- André Raspaud (LaBRI, France),
- Jozef Širáň (Slovak University of Technology, Bratislava, Slovakia), and
- Pavel Valtr (Charles University, Prague, Czech Republic).

I would like to thank for help with the organization of this conference to Tomáš Gavenčiak and Petr Gregor. But also I shall stress that also several other colleagues willingly assisted us with this effort. I am much obliged to all of them. I would like to emphasize that the conference would not be possible to arrange without support of the Centre of Excellence - Institute for Theoretical Computer Science (CE-ITI), of the Department of Applied Mathematics of the Charles University, of the Czech Mathematical Society and of the European Training Centre in Litomyšl.

Dear colleagues, I wish you to bring home several new ideas for your further research and also only grateful memories on this week spent in Litomyšl.

Jiří Fiala

Předmluva

Dovolte mi, abych vás co nejsrdečněji přivítal v Litomyšli, malebném městě na pomezí Čech a Moravy, městě proslulém svými historickými a kulturními skvosty, pro něž bylo právem zařazeno na seznam kulturního dědictví UNESCO.

Je všeobecně známo, že Litomyšl je rodištěm Bedřicha Smetany, že na zdejším gymnáziu čtrnáct let učil Alois Jirásek, a že zde pobývali známé osobnosti kulturního života jako například Božena Němcová, Julius Mařák nebo Josef Váchal.

Méně se ví, že se zde 16. dubna 1848 narodil i prof. Karel Zahradník, český matematik, který značnou část svého vědeckého života strávil v Chorvatsku na Záhřebské universitě a později byl v roce 1899 prvním rektorem Vysokého učení technického v Brně. Ještě před svým odchodem do Chorvatska se v Praze podílel na založení Jednoty českých matematiků, odborného spolku, který právě letos slaví 150. výročí svého založení.

Naše konference o teorii grafů a kombinatorice nemá tak dlouhou tradici jako Jednota českých matematiků a fyziků, ovšem od první konference, která se uskutečnila v roce 1961 v Liblicích se setkáváme již po sedmačtyřicáté. U zrodu této tradice stál i prof. Miroslav Fiedler, a jsem velmi rád, že zde jej letos mohu přivítat.

Doufám, že se i letošní konference zdaří k vaší spokojenosti. K tomu jistě přispějí zvané přednášky, které přednesou

- Hajo Broersma, (University of Twente, Nizozemí),
- Katarína Cechlárová (UPJŠ Košice),
- Tomáš Kaiser (ZČU Plzeň),
- Jan Obdržálek (MU Brno),
- André Raspaud (LaBRI, Francie),
- Jozef Širáň (STU Bratislava) a
- Pavel Valtr (UK Praha).

Mé poděkování za pomoc s uspořádáním této konference patří zejména Tomáši Gavenčiakovi a Petru Gregorovi. Nutno však dodat, že nejen oni, ale i řada dalších ochotně přispěla svou pomocí. Všem patří můj dík. Rád bych zdůraznil, že by konferenci nebylo možné uspořádat bez podpory Centra excelence — Institutu teoretické informatiky, Katedry aplikované matematiky MFF UK, České matematické společnosti, sekce JČMF a Evropského školicího centra v Litomyšli.

Vážení kolegové, milé kolegyně, přeji vám, abyste si z konference odvezli řadu zajímavých námětů pro vaše další bádání a jen samé příjemné vzpomínky na tento týden strávený v Litomyšli.

Jiří Fiala

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Part I

Invited Talks

Subgraph conditions for hamiltonian properties of graphs

Hajo Broersma

University of Twente, The Netherlands

We will discuss the state of the art of existing and recently obtained subgraph conditions that are imposed on a graph for guaranteeing that the graph contains a Hamilton cycle, a Hamilton path, or for possessing similar properties related to hamiltonicity. One type of subgraph conditions, that has been a popular research subject since the 1980s, consists of forbidden subgraph conditions, i.e. imposing that certain (typically quite small) graphs are excluded as an induced subgraph, with a major stream of research focussing on claw-free graphs. Instead of forbidding certain induced subgraphs, one can allow these subgraphs while imposing other structural conditions on their vertices, e.g. degree or degree sum conditions. This has led to results that generalize the existing forbidden subgraph results as well as the early degree condition results for hamiltonian properties due to Dirac and Ore.

Housing markets and graphs

Katarína Cechlárová

P. J. Šafárik University, Košice

A formal notion of a housing market was introduced by Shapley and Scarf in 1974. An instance of a housing market is a pair $\mathcal{M} = (A, \mathcal{P})$, where A is a set of agents, each of them owns one unit of some indivisible good (traditionaly called a house) and wishes to end up again with just one house, possibly more preferable than his original one. $\mathcal{P}$ is the collection of preference lists of agents, while preferences of agent a are given as a linear ordering $P(a)$ of A. A representation of housing markets in the form of digraphs helped to solve many problems that occur in this setting.

There are many real situations that can be modelled by housing markets, the most important ones are the exchange of kidneys for transplantation, housing exchange fairs in Beijing and in recent years several Internet networks for exchanges of holiday houses, books, CD's, spare shoes etc. In these markets the exchanges are performed without any monetary transfers, so various notions of optimality come into consideration.

We shall deal with Pareto optimality and Strong Pareto optimality, but our main focus will be the Core and the Strong core. We present the classical Top Trading Cycles algorithm (TTC for short) of Gale that finds a core exchange in each housing market. If preferences of agents are strict, the TTC permutation belongs to the strong core as well, however, if preferences contain ties, strong core may be empty and we present another algorithm to decide its nonemptyness.

A complete description of the core is not known. We formulate several questions about its structure that turned out to be hard. For a few specially formed geometric markets we present some structural observations. In conclusion we mention some modifications of the basic model and open questions connected with them.

Connectivity, factors and the method of iterated partitions

Tomáš Kaiser

University of West Bohemia, Plzeň

The *method of iterated partitions* was developed as a tool for problems involving graph factors with suitable connectivity properties. In this talk, we outline the method and use it to give a short proof [2] of the characterization of graphs with k disjoint spanning trees of Tutte and Nash-Williams [5, 6]. We then review applications of the method which yield the following results:

- 5-connected line graphs with minimum degree at least 6 are Hamilton-connected [3],
- 3-connected, essentially 9-connected line graphs are Hamilton-connected [4],
- every 5-edge-connected graph admits a spanning tree with 2-edge-connected complement [1].

For each of the applications, we highlight a characteristic feature, such as the use of hypergraphs or the discharging method. Related ongoing research will also be discussed.

The talk is partly based on joint work with Petr Vrána.

Reference

[1] T. Kaiser, Spanning trees with 2-edge-connected complement, in preparation.

[2] T. Kaiser, A short proof of the tree-packing theorem, Discrete Math. 312 (2012), 1689–1691.

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[4] T. Kaiser and P. Vrána, The hamiltonicity of essentially 9-connected line graphs, in preparation.

[5] C. St. J. A. Nash-Williams, Edge-disjoint spanning trees of finite graphs, J. London Math. Soc. (1961), 445–450.

[6] W. T. Tutte, On the problem of decomposing a graph into n connected factors, J. London Math. Soc. 36 (1961), 221–230.

MSO **model checking lower bounds**

Jan Obdržálek

Masaryk University, Brno

Among algorithmic meta-theorems a special place belongs to the famous theorem of Courcelle, which states that that any graph problem definable in monadic second-order logic with edge-set quantifications (MSO_2) is decidable in linear time on any class of graphs of bounded tree-width (in other words the MSO_2 model-checking problem is fixed parameter tractable (FPT) w.r.t. tree-width). Only recently Kreutzer and Tazari [2] proved a corresponding lower bound, by showing that MSO_2 model-checking is not even in XP (and hence not in FPT), w.r.t. the formula size as parameter, for graph classes that are subgraph closed and whose tree-width is poly-logarithmically unbounded – assuming the Exponential Time Hypothesis (ETH) holds.

We show a closely related result: That even MSO_1 model-checking (where we cannot quantify over sets of edges) with a fixed set of vertex labels is not in XP, w.r.t. the formula size as parameter, for graph classes which are again subgraph-closed and whose tree-width is poly-logarithmically unbounded. Here we assume that non-uniform ETH holds, which also allows us to present a streamlined proof avoiding the complex machinery used by Kreutzer and Tazari.

Our result has an interesting consequence in the realm of digraph width measures: Strengthening the recent result [1], we show that no subdigraph-monotone measure can be algorithmically useful, unless it is within a poly-logarithmic factor of (undirected) tree-width.

Joint work with Robert Ganian, Petr Hliněný, Alexander Langer, Peter Rossmanith, and Somnath Sikdar.

Reference

[1] R. Ganian, P. Hliněný, J. Kneis, D. Meister, J. Obdržálek, P. Rossmanith, and S. Sikdar. Are there any good digraph width measures? In *IPEC'10*, volume 6478 of *LNCS*, pages 135–146. Springer, 2010.

[2] S. Kreutzer and S. Tazari. Lower bounds for the complexity of monadic second-order logic. In *LICS'10*, pages 189–198, 2010.

$(d_1, d_2, \ldots, d_k)$-colorings of graphs

André Raspaud

Université de Bordeaux - LaBRI, France

A graph G is called *improperly* $(d_1, d_2, \ldots, d_k)$*-colorable*, or just $(d_1, d_2, \ldots, d_k)$*-colorable* if the vertex set of G can be partitioned into subsets $V_1, \ldots, V_k$ such that the graph $G[V_i]$ induced by the vertices of V_i has maximum degree at most d_i for all $1 \leq i \leq k$. This notion generalizes those of proper k-coloring (when $d_1 = \ldots = d_k = 0$) and d-improper k-coloring (when $d_1 = \ldots = d_k = d \geq 1$). Proper and d-improper colorings have been widely studied. Under this terminology, the Four Color Theorem says that every planar graph is $(0, 0, 0, 0)$-colorable. In this talk we will give a short survey of the (i, j)-coloring and new results concerning the (i, j, k)-colorings.

Algebraic methods in the degree-diameter problem

Jozef Širáň

The Open University, Milton Keynes, United Kingdom and Slovak University of Technology, Bratislava

For given positive integers $d \geq 3$ and $k \geq 2$ let $n(d,k)$ be the largest order of a graph of maximum degree d and diameter k. Determination of $n(d,k)$ and classification of the corresponding graphs is known as the *degree-diameter problem*. Despite considerable interest that generated more than a hundred of papers on the topic, only seven exact values of $n(d,k)$ in the above range for d and k are known! On the other hand, a number of highly non-trivial bounds on $n(d,k)$ are available and most of these have been proved by algebraic techniques involving spectral theory and group theory.

An easy upper bound on $n(d,k)$ is the *Moore bound*, equal to the order $M(d,k)$ of a central tree of maximum degree d and radius k. It is surprisingly hard to prove (by spectral methods) that $n(d,k) \leq M(d,k) - 2$ for all d,k as above except for $k = 2$ and $d = 3$, 7 and, possibly, 57; this is the only general upper bound on $n(d,k)$ we have. All known lower bounds on $n(d,k)$ have been obtained by constructions from certain underlying algebraic structures (groups, in most cases). These, however, asymptotically approach the Moore bound only rarely, leaving a huge gap between the two bounds in general and offering a number of opportunities for further research.

In the talk we will give a brief survey of algebraic methods that have been used to prove significant results in the degree-diameter problem. Our discussion will also include restrictions of the degree-diameter problem to vertex-transitive and Cayley graphs and analogous questions for directed graphs.

Visibility and invisibility graphs

Pavel Valtr

Charles University, Prague

There are more types of visibility graphs. The vertices are usually points of some region in the plane, and two points are connected by an edge if the segment connecting them intersects none of given obstacles. Sometimes it is natural to consider the complement of the visibility graph, called the invisibility graph. Another concept concerns visibility graphs of point sets in the plane, where the vertex set of the visibility graph coincides with the set of obstacles - thus, two points are connected by an edge if there is no other point on the segment connecting these two points. I will talk about various interesting questions and results for visibility graphs of different types.

Part II

Regular Contributions

A construction of Cayley graphs of diameter two

Marcel Abas

Slovak University of Technology, Bratislava

In the graph theory, a very known problem is the *degree-diameter problem*, which deals with determining of the larger order $n(d,k)$ of a graph with given maximum degree $d \geq 2$ and diameter k. The *Moore bound* on the order is $n(d,k) \leq 1+d+d(d-1)+\ldots d(d-1)^{k-1}$. This number gives for the diameter $k=2$ the upper bound $n(d,2) \leq d^2+1$ and the bound is achieved only for degrees $d=2,3,7$, and possibly for $d=57$. For general graphs there is a construction with $n(d,2) \geq d^2-d+1$ if $d-1$ is a prime power. For vertex transitive graphs, the best lower bound is $v(d,2) \geq \frac{8}{9}(d+\frac{1}{2})^2$ for $d=\frac{1}{2}(3q-1)$ where $q \equiv 1 \pmod 4$ is a prime power. Finally, the best lower bound for Cayley graphs is $c(d,2) \geq \frac{1}{2}(d+1)^2$ for $d=2q-1$ where q is a prime power. We note that for Cayley graphs there is a *folklore* bound $c(d,2) \geq \lfloor\frac{d+2}{2}\rfloor\lceil\frac{d+2}{2}\rceil$ valid for all degrees $d \geq 2$.

In this contribution we present a construction for Cayley graps with $c(d,2) \geq \frac{16}{49}d^2$ valid for all degrees $d \geq 6$.

Keywords: Cayley graph, Moore bound.

Induced subarrays of Latin squares without repeated symbols

Julian Abel, Nick Cavenagh, and Jaromy Kuhl

University of Waikato, New Zealand

Given a Latin square of even order n, partitions of the rows and column into pairs induce $n^2/4$ 2×2 subarrays. We pose the following problem: For large enough n, is it possible to find such a partition so that each induced subarray contains no repeated symbol? We present some partial results.

Locally constrained homomorphism with bounded parameters

Isolde Adler, Steven Chaplick, Jiří Fiala, Pim van't Hof,
Daniël Paulusma, and Marek Tesař

Charles University, Prague

A homomorphism from graph G onto graph H is an edge preserving mapping of vertices of G onto vertices of H. We say that homomorphism $f\colon V(G) \to V(H)$ is locally injective (surjective, bijective) if restriction of f to the neighborhood of any vertex u of G maps these vertices injectively (surjectively, bijectively) to the neighborhood of $f(u)$ in H. Then we can define a problem LIHom (LSHom, LBHom) as a problem of deciding if there exists a locally injective (surjective, bijective) homomorphism from a given graph G to the given graph H.

It is well known that all three problems are NP-complete. We study the computational complexity of the problems LIHom, LSHom, and LBHom where both G and H are parametrized by some graph parameter. For example these problems are NP-complete if both graphs G and H have bounded degree or bounded path-width. We show that if G and H have bounded both degree and tree-width then all these problems are polynomially solvable.

Archimedean operations and vertex-transitive maps

Antonio Breda d'Azevedo, Domenico Catalano, Ján Karabáš, and Roman Nedela

Matej Bel University, Banská Bystrica

It is well-known that in classical crystallography, all Archimedean solids can be obtained from Platonic solids applying few operations. These operations are determined by describing local changes of the respective spherical maps forming the 2-skeletons of the considered solids. In geometry they are known as formation of the dual map, truncated map, medial map, dual of the barycentric subdivision and others. All these operations have following features:

1. they preserve the underlying surface,
2. the resulting map admits the action of automorphism group of the original map.

We give a general definition of an operation satisfying the above properties and defined on the class of all maps. In addition to the above-mentioned, the complex structure of the Riemann surface associated with the map is preserved by the operations as well.

In what follows, we define a restricted subclass of these operations, giving rise to vertex-transitive maps when applied on the family of regular maps. In analogy with the classic we call these operations *Archimedean operations*. We show that independently on the genus of the underlying surface there are 9 well-defined Archimedean operations with the following property. Given list of all regular maps of fixed genus $g > 1$, applying these nine operations on the list we obtain all non-degenerate vertex-transitive maps of genus g admitting actions of a quotient of a triangle group of genus g. Application of Archimedean operation on a regular map M yields in the five cases a vertex-transitive map, in the other four cases the resulting map is vertex-transitive provided the original regular map admits some additional external symmetries. In this case the automorphism group G of M acts with two orbits on vertices and the action of an additional symmetry makes the map vertex-transitive. On the other hand, we prove that if a vertex-transitive map M is non-degenerate and admits an action of a triangle group then it acts with at most nine orbits on vertices and M is either regular or it comes from some regular map applying one of the above 9 operations. The statement includes the classical case when the underlying surface is the sphere, in particular, it follows that the classical Archimedean solids can be constructed from the five Platonic solids and the infinite family of cycles on the sphere. Our result generalizes an earlier result by Singerman classifying operations on regular maps preserving the underlying Riemann surface.

Brown graphs revisited

Martin Bachratý

Comenius University, Bratislava

Let $\mathcal{P}(q)$ be the standard projective plane over a field $F = GF(q)$, q a prime power. Vertices of the *Brown graph* $B(q)$ are the q^2+q+1 points of $\mathcal{P}(q)$ represented by projective triples over $GF(q)$. Two distinct vertices $a = [a_1, a_2, a_3]$ and $b = [b_1, b_2, b_3]$ are adjacent in $B(q)$ if and only if $ab^T = a_1b_1 + a_2b_2 + a_3b_3 = 0$.

Brown graphs are a basis for construction of the largest currently known graphs of diameter 2 and given maximum degree, but their properties have not been studied so far. In our work we investigate the structure and properties of Brown graphs in detail, including determination of their automorphism groups.

Construction of 4-regular graphs with circular chromatic index close to 4

Barbora Candráková

Comenius University, Bratislava

The circular chromatic index of a graph G is the infimum of all rational numbers p/q, such that there exists a circular p/q-edge-coloring of the graph G.

It is known that the value of the circular chromatic index is rational and is always attained for finite graphs. Circular edge-colorings can be viewed as a refinement of the edge-coloring, since $\chi'(G) = \lceil \chi'_c(G) \rceil$ where $\chi'(G)$ is the chromatic index of G.

In view of the celebrated Vizing's theorem, circular edge-colorings are especially interesting for d-regular graphs with chromatic index $d+1$, that is d-regular class 2 graphs. While circular edge-colorings of cubic graphs have been extensively studied, very little is known about circular edge-colorings of d-regular graphs with $d \geq 4$. We present a construction of 4-regular graphs with circular chromatic index equal to $4 + 2/r$ for each integer r greater than 8. So far, no similar result has been known for 4-regular graphs.

Facial parity edge colouring of bridgeless plane graphs

Július Czap, Stanislav Jendrol', František Kardoš, and Roman Soták

P. J. Šafárik University, Košice

A *facial parity edge colouring* of a connected bridgeless plane graph is such an edge colouring in which no two face-adjacent edges receive the same colour and, in addition, for each face f and each colour c, no edge or an odd number of edges incident with f are coloured with c. Let $\chi'_p(G)$ denote the minimum number of colours used in a such colouring of G. In this paper we prove that $\chi'_p(G) \leq 20$ for any 2-edge-connected plane graph G. In the case when G is a 3-edge-connected plane graph the upper bound for this parameter is 12. For G being 4-edge-connected plane graph we have $\chi'_p(G) \leq 9$. On the other hand we prove that some bridgeless plane graphs require at least 10 colours for such a colouring.

Hypomorphisms of graphs, their generalisations and applications

Peter Czimmermann

University of Žilina

A hypomorphism from a graph $G = (V_G, E_G)$ to a graph $H = (V_H, E_H)$ is a bijection $f : V_G \to V_H$ such that for all vertices $v \in V_G$ it holds that the vertex deleted subgraph $G - v$ is isomorphic to the vertex deleted subgraph $H - f(v)$. Hypomorphisms are closely related to the reconstruction (Kelly-Ulam) conjecture, since one of the formulations of this conjecture is: If two graphs are hypomorphic, then they are also isomorphic.

In our contribution, we consider several generalisations of the hypomorphisms (especially $(k, n - k)$-hypomorphisms) and their applications. We focus mainly on the following results: i) Graphs G and H are isomorphic if and only if there is an intersection-preserving $(k, n - k)$-hypomorphism from G to H. ii) Set of all intersection-preserving $(k, n - k)$-hypomorphisms on G forms a group isomorphic to the automorphism group of G. iii) Extension of these results to hypergraphs. iv) Application in the proof that almost every hypergraph is reconstructible.

Radio labeling of distance graphs

Roman Čada, Jan Ekstein, Přemysl Holub, and Olivier Togni

University of West Bohemia, Plzeň

Let k be a positive integer and G a simple undirected graph. A *k-radio labeling* f of G is an assignment of non negative integers to the vertices of G such that $|f(u) - f(v)| \geq k + 1 - \mathrm{dist}_G(u, v)$, for every two distinct vertices u and v of G. The span of the function f, denoted by $\mathrm{rc}_k(f)$, is $\max\{f(x) - f(y) : x, y \in V(G)\}$. The *$k$-radio chromatic number* $\mathrm{rc}_k(G)$ of G is the minimum span among all radio k-labelings of G.

In this work, radio labelings of distance graphs are studied and some upper and lower bounds are given for distance graphs with distance set $\{1, 2, \ldots, t\}$, $\{1, t\}$ and $\{t - 1, t\}$, where $t \geq 2$ is a positive integer. Moreover the upper and lower bounds are asymptotically equal in any of the mentioned cases.

On a circumference of 2-factors in claw-free graphs

Roman Čada and Shuya Chiba

University of West Bohemia, Plzeň

Matthews and Sumner proved in 1985 that in a 2-connected claw-free graph (i.e. a graph with no induced $K_{1,3}$) there is a cycle of length at least $\min\{2\delta(G)+4, n\}$ ($\delta(G)$ is the minimum degree of G). We show a generalization of this result: in a 2-connected claw-free graph with minimum degree at least 7 there is a 2-factor with a longest cycle of length at least $\min\{2\delta(G)+4, n\}$. We also discuss variations on this result.

Connected even factors in the square of a graph

Jan Ekstein, Shinya Fujita, and Kenta Ozeki

University of West Bohemia, Plzeň

Let G be a simple undirected graph. The *square of* G is the graph G^2 with the same vertex set as G, in which two vertices are adjacent if and only if their distance in G is at most 2. A famous result of Fleischner states that the square of G of any 2-connected graph is hamiltonian. We study a generalization of hamiltonian cycle on $[2, 2s]$-factors. A $[2, 2s]$-factor in a graph G is a connected even factor with maximum degree at most 2s. We show that the square of G of any 2-edge connected graph has a $[2, 4]$-factor.

Constrained homomorphism orders

Jiří Fiala, Jan Hubička, Yangjing Long

Charles University, Prague

For given graphs G and H a *homomorphism* $f : G \to H$ is a mapping $f : V_G \to V_H$ such that $(u, v) \in V_G$ implies $(f(u), f(v)) \in V_H$. If there exists a homomorphism $f : G \to H$ we write $G \to H$. It is well known that $\to$ (seen as binary relation) induce a quasi order on the class of all finite graphs. The equivalence classes contains up to isomorphism unique minimal representative, the graph core. The homomorphism order is partial order on graph cores induced by $\to$.

It is a non-trivial result that every countable partial order can be found as a suborder of the homomorphism order. The initial result has been proved in even stronger setting of category theory. Subsequently it has been shown that even more restricted classes of graphs and similar structures (such as homomorphisms of set systems oriented trees, oriented paths, partial orders and lattices) admit this universality property.

Pair of graphs (G, H) such that $G < H$ and there is no G', $G < G'H$ is called *gap*. All gaps in the homomorphism order has been characterized. In fact the only gap is (K_1, K_2) and thus the partial order is dense when this pair is removed. The structure of gaps is more rich on the homomorphism order of directed graphs and is closely related to the notion of homomorphism dualities.

Several variants of graph homomorphism also have been studied (such as locally constrained homomorphisms, full homomorphisms or surjective homomorphisms). We consider partial order induced by these homomorphisms and ask the same questions — prove or disprove the universality of the partial order and characterize gaps. We also show a new and easier proof of the universality of homomorphism order that is easier to apply in this setting.

Finding contractions in claw-free graphs

Jiří Fiala, Marcin Kamiński, and Daniël Paulusma

Charles University, Prague

We consider the problems that are to test whether a given host graph contains a fixed target graph as a contraction.

We define a pileous clique as a graph whose vertices of degree at least two form a clique.

Theorem 1 *If H is a fixed pileous clique, then H-*Contractibility *is solvable in polynomial time on claw-free graphs.*

Let $F = L(G)$ be the line graph of a graph G. Without loss of generality we may assume that F is different from K_3, hence the graph G is uniquely determined by F. Observe that F contains P_k as a contraction if and only if the edges of G can be partitioned into k nonempty classes called *color classes*, $E_1, \ldots, E_k$, such that each color class E_i induces a connected subgraph in G and moreover, an edge of some color i may only intersect edges of color 1 or 2 if $i = 1$, edges of color $i-1$, i, or $i+1$ if $2 \leq i \leq k-1$ and edges of color $k-1$ or k if $i = k$. We call this problem the k-Edge Partition problem. Clearly, P_k-Contractibility on line graphs and k-Edge Partition are polynomially equivalent.

Theorem 2 *The k-*Edge Partition *problem is* NP*-complete for $k = 7$.*

Corollary 1 *The P_7-*Contractibility *problem is* NP*-complete for line graphs.*

We leave as an open problem to determine the computational complexity of the P_k-Contractibility problem for $k = 5$ and 6.

Hamilton Cycle matrices

Miroslav Fiedler

Prague

Hamilton Cycle matrices, shortly HC-matrices, are square matrices the digraph of non-zero entries of which contains a Hamilton cycle (a cycle passing through all vertices). If such cycle is unique, we say that the matrix is a UHC-matrix (uniquely Hamiltonian cycle matrix). A simple example of a UHC-matrix is the (well known among numerical analysts) Hessenberg matrix, matrix having in the lower triangular part non-zero entries just in the first subdiagonal, under the condition that the upper-right corner entry is also different from zero.

In a series of papers, the author (partially with F. J. Hall) studied matrices obtained by multiplication of simpler matrices, each differing from the identity matrix by one diagonal block, with some restrictions. It turned out the resulting products have intriguing properties. All of them (with fixed factors) have the same spectrum independently of their ordering, they have certain zero - nonzero shapes, certain submatrices of lower rank, etc. The usual companion matrix of a polynomial belongs to such kind of matrices, and this fact led to the discovery of other simple companion matrices.

We shall show that if all the mentioned diagonal blocks are HC-matrices (resp., UHC-matrices), then the product is also a HC-matrix (UHC-matrix). And this works for all product matrices, independently of their ordering.

Faster than Courcelle's theorem on shrubs

Jakub Gajarský and Petr Hliněný

Masaryk University, Brno

$$\mathcal{O}\left(|V(G)| \cdot 2^{\left.2^{\cdot^{\cdot^{2^k}}}\right\}\text{quantifier d.}}\right) \qquad \text{vs.} \qquad \mathcal{O}\left(|V(G)| \cdot 2^{\left.2^{\cdot^{\cdot^{2^r}}}\right\}\text{tree-depth}}\right)$$

Famous Courcelle's theorem claims FPT solvability of any MSO_2-definable property in linear FPT time on the graphs of bounded tree-width (alternatively, of MSO_1 on clique-width by Courcelle-Makowsky-Rotics). A drawback of this powerful algorithmic metatheorem is that its runtime has a nonelementary dependence on the quantifier alternation depth of the defining formula (above left). This is indeed unavoidable in full generality (even on trees) as shown by Frick and Grohe. We show a new approach to this problem, giving an MSO model checking algorithm on trees of bounded height in FPT with elementary dependence on the formula; actually, we "trade" a nonelementary dependence on the formula for a nonelementary dependence on the height (above right). This implies a faster (than Courcelle's) algorithm for all MSO_2-definable properties on the graphs of bounded tree-depth, and similarly a faster algorithm for all MSO_1-definable properties on the classes of bounded shrub-depth—which is a new notion defined just recently in collaboration with Ganian, Obdržálek, Nešetřil, Ossona de Mendez, and Ramadurai.

Deciding first order logic properties of matroids

Tomáš Gavenčiak, Daniel Král', Sang-il Oum

Charles University, Prague

Classes of graphs with bounded tree-width play an important role both in structural and algorithmic graph theory. As shown by Courcelle, monadic second order formulas can be effectively decided on graphs of bounded tree-width.

Matroids are combinatorial structures generalizing graphs and linear independence. Branch-width is a natural width parameter for matroids, and for graphical matroids differs by at most multiplicative constant from their tree-width. Hlineny and Oum have generalized Courcelle's theorem to matroids of bounded branch-width represented over a fixed finite field.

Frick and Grohe introduced a notion of graph classes with locally bounded tree-width and established that every first order logic property can be decided in almost linear time ($O(n^{1+\varepsilon})$ for any fixed ε) in such a graph class. Here, a graph class $\mathcal{C}$ has locally bounded tree-width when there is a function f such that for any graph $G \in \mathcal{C}$ and any $H \subseteq G$, the tree-width of H is at most f of the diameter of H.

We introduce an analogous notion for matroids (called locally bounded branch-width) and show the existence of a fixed parameter algorithm for first order logic properties in classes of regular matroids with locally bounded branch-width. In order to obtain this result, we show that the problem of deciding the existence of a circuit of length at most k containing two given elements is fixed parameter tractable for regular matroids.

Subcube isoperimetry and power of coalitions

Petr Gregor

Charles University, Prague

We determine the minimal number of d-dimensional subcubes with a vertex in A and simultaneously a vertex not in A, over all sets A of k vertices in the n-dimensional hypercube. This extends a classical result of Harper on the edge-isoperimetric problem in the hypercube. We study properties of extremal sets with means of harmonic analysis of corresponding Boolean functions. Applications range from labeling vertices of the hypercube so that the total maximal deviation of labels on subcubes is minimized, to study of influence of coalitions in simple voting games via their Banzhaf power index.

On the palette index of a graph

Mirko Horňák, Rafał Kalinowski, Mariusz Meszka, and Mariusz Woźniak

P. J. Šafárik University, Košice

Let G be a finite simple graph, C a set of colours and $\varphi : E(G) \to C$ a proper edge colouring of G. The *palette* of a vertex $v \in V(G)$ (with respect to φ) is the set $\{\varphi(e) : e \ni v\}$ of colours of edges incident with v. The *palette index* of the graph G is the minimum number of (distinct) palettes in a proper edge colouring of G. The palette index is determined for complete graphs and for cubic graphs.

Chiral regural maps of given type

Veronika Hucíková

Slovak University of Technology, Bratislava

A *map* is a graph embedded in surface in such way that every face is homeomorphic to the open disk. We are interested in highly symmetric maps, which are called *regular maps*. In this family of maps we are looking for maps which has no orientation reversing automorphism, but still have maximum possible number of the orientation preserving automorphisms. Taking motivation from chemistry, this maps have become known as *chiral regular maps*.

The *type of embedding* is a pair (k, m), where k is less common multiple of degrees of vertices and m is less common multiple of size of faces. In case of the regular map all vertices have some degree and all faces have some size, so k is the degree of each vertex and m the size of each face. Our aim is to prove, that (up to a finite number of exceptions) there exists a chiral regular map of type (k, m) for every pair (k, m) such that $1/k + 1/m < 1/2$.

For this we introduce following. Let M be a map. The edges and the semi-edges of the previous graph are now continuous images of the closed interval $[0, 1]$ and the half-open interval $[0, \frac{1}{2})$, respectively. The image of half-open interval $[0, \frac{1}{2})$ and $(\frac{1}{2}, 1]$ are *darts* and form set of the darts D of map M. We use to describe the map M as a triple $(D; R, L)$, where L and R are permutation on D. The *dart-flip* L letting $xL = x$ if x is a semi-edge, and $xL = y$ if the darts x, y form an edge that is not a semi-edge. The *rotation* R determinates order of darts around each vertex.

The permutations R and L generate the *group of monodromy* $Mod(M) = \langle R, L\rangle$ of M. If the group of monodromy of M is $S_{|D|}$ or $A_{|D|}$, then we say, that M is *strongly chiral*. If $M = (D; R, L)$ is strongly chiral, then canonical regular cover, the map $\widetilde{M} = (\langle R, L\rangle; R, L)$, is a regular chiral map as long as $|D| > 6$. This reduces our problem, to find strongly chiral (not necessary regular) maps of given type.

Smallest graphs of given degree and diameter

Martin Knor and Jozef Širáň

Slovak University of Technology, Bratislava

The degree-diameter problem consists in finding the largest order $N(d,k)$ of a (regular) graph of maximum degree d and diameter k. In some variants of this problem we restrict ourselves to vertex-transitive or Cayley graphs. In this talk we consider an opposite problem. Namely, we find the smallest order $n(d,k)$ of a regular (vertex-transitive or Cayley) graph of degree d and diameter k, and in all three cases of regular, vertex-transitive and Cayley graphs, we give a complete solution.

Genus of abelian groups with Z_3 factor

Michal Kotrbčík and Tomaž Pisanski

Comenius University, Bratislava

The genus of a group is the minimum among genera of Cayley graphs of the group. While the genus of most abelian groups not having Z_3 factor is known, the groups containing Z_3 cause tremendous difficulties. In this talk we focus mainly on the genus of G_n, the cartesian product of n triangles, which is the Cayley graph of the direct product of n copies of Z_3. Using a lifting method we present a general construction of a low-genus embedding of G_n using a low-genus embedding of G_{n-1}. Our method provides currently the best upper bound on the genus of G_n for all $n \geq 5$. We report results obtained by computer search, which include improving the upper bound on the genus of G_4 to 39, complete genus distribution of G_2, and more than 200 nonisomorphic genus embeddings of G_3. Additionally, we discuss further applications of our methods and a relationship among the genus of G_n, the genus of related Cartesian products, and variations of the genus of a group.

Small subgraphs in triangle free strongly regular graphs

Kristína Kováčiková

Comenius University, Bratislava

Strongly regular graphs represent an important class of graphs which stands somewhere between highly symmetric and randomly generated graphs. Thanks to their remarkable properties, they found an application in many areas of science, for example in cryptography, group theory or theoretical chemistry.

A graph G with parameters (n, k, μ, λ) is strongly regular, SRG(n, k, μ, λ), iff it is k-regular on n vertices and it holds that:

1. Any two adjacent vertices have exactly μ common neighbors.

2. Any two nonadjacent vertices have exactly λ common neighbors.

The most popular example of SRG is Petersen graph, whose parameters are $(10, 3, 0, 1)$. The conditions above may be considered too strict and give the impression that we have enough information for construction of SRG, but in general this could be really hard problem. This is the reason, why it is useful to look for properties of SRG, omitting the construction itself. We will show that it is possible to determine the number of small induced subgraphs of SRG with using only the parameters n, k, λ, μ. The methods that we want to present are usable for all sets of parameters. Despite of this we will focus only on cases where $\lambda = 0$.

Decomposition of complete graphs into compact subgraphs

Petr Kovář, Tereza Kovářová, Michal Kravčenko, and Dalibor Lukáš

VŠB – Technical University of Ostrava

When solving large systems of equations it is natural to decompose the corresponding large matrices into smaller (sub)matrices and parallelize the computation on a cluster with many nodes. If the parallel machine uses distributed memory, further requirements on the decomposition arise.

For simplicity let A be a large full matrix with $n \times n$ blocks B_{ij}. We want to choose n sets $C_1, C_2, \ldots, C_n$ of blocks so that each block B_{ij} of A belongs to some set C_k and the maximum number of different block subscripts in each C_k is as small as possible. We rephrase the task in the language of graph decompositions and for certain values of n also as a number theory problem of perfect difference sets.

We present some constructions of decompositions of complete graphs K_n into small dense graphs that can be used to solve the problem above. The decompositions have been implemented and successfully tested for fast BEM matrices of size up to millions distributed to hundreds of nodes.

Keywords: graph decomposition, rho-labeling, perfect difference sets.

Handicap labelings of regular graphs

Petr Kovář and Tereza Kovářová

VŠB – Technical University of Ostrava

Let $G = (V, E)$ be a simple graph on n vertices. A bijection $f : V \to 1, 2, \ldots, n$ is called a *handicap labeling* if there exists an integer ℓ such that $\sum_{v \in N(u)} f(v) = \ell + f(u)$ for all $u \in V$, where $N(u)$ is the set of all vertices adjacent to u. A graph that admits a handicap labeling is called a *handicap graph*. Handicap graphs can be used for scheduling incomplete round robin tournaments in which the sum of strengths of opponents of each team is increasing with the strength of the team. We present a construction of handicap labelings for r-regular graphs where $r \equiv 1, 3 \pmod 4$.

Keywords: handicap labeling, incomplete tournament, scheduling.

On q-chromatic function and graph isomorphism

Martin Loebl and Jean-Sébastien Sereni

Charles University, Prague

We show some observations related to a conjecture (of Loebl) that q-dichromate distinguishes non-isomorphic chordal graphs.

Dividing the edges equitably

Robert Lukoťka and Ján Mazák

University of Trnava

We introduce an optimization problem consisting of distributing value from the edges to the vertices. We prove certain general results and show their applications in constructing graphs with given circular chromatic index.

Each edge e of a graph G has value 1. For every edge e we want to divide its value between two vertices incident to e in ratio at most $r \geq 1$. Such a division will be called an r-edge-division. The value of a vertex is the sum of values the vertex obtained from its incident edges. A vertex of highest value is called a rich vertex and a vertex of smallest value is called a poor vertex. For a graph G we define $\pi_r(G)$ ($\rho_r(G)$) to be the largest (smallest) possible value of a poor (rich) vertex among all r-edge-divisions in G.

Primary aim of the talk will be to explore the range of $\pi_r(G)$ (and $\rho_r(G)$). We show that the range is the same for both invariants. The main result of the talk is almost determining the range of $\pi_2(G)$ and $\rho_2(G)$. We show that the range of $\pi_2(G)$ is

$$\mathbb{Q} \cap (\{0, 1/2, 2/3, 1, 1 + 1/3\} \cup [1 + 1/2, \infty) \cup A).$$

The set A contains many values between 1 and $1 + 1/3$, however a value x between 1 and $1+1/3$ is not in A when $x = 1+s/(3t)$ and $(t \bmod s) > 2/3\, s$. We mention the consequences of our results for the circular chromatic index graphs of certain class of graphs.

Circuit covers in signed graphs and nowhere-zero flows

Edita Máčajová, André Raspaud, Edita Rollová, and Martin Škoviera

Comenius University, Bratislava

A circuit cover of a graph G is a collection $\mathcal{C}$ of circuits such that each edge of G belongs to at least one circuit from $\mathcal{C}$. There is a natural analogue of this concept for signed graphs, graphs where each edge is either positive or negative. As suggested by matroid theory, a signed circuit is either a single circuit with an even number of negative edges, or a pair of disjoint circuits with an odd number of negative edges each, joined with a path. It can be shown that a signed graph has a circuit cover if and only if it admits a nowhere-zero integer flow. In the talk we will mention several bounds on the length of a shortest circuit cover in a signed graph G, depending on the existence of a nowhere-zero k-flow in G. In particular we show that a signed graph G that admits a nowhere-zero 2-flow has a circuit cover with total length at most $\frac{4}{3} \cdot |E(G)|$. This bound is tight for infinitely many graphs.

Nowhere-zero flows in signed complete and complete bipartite graphs

Edita Máčajová and Edita Rollová

Comenius University, Bratislava

A signed graph $G = (V, E, \Sigma)$ is a graph with vertex set V, edge set E and a mapping $\Sigma : E \to \{+1, -1\}$. Thus each edge becomes either positive or negative. To get an orientation of an edge e of a signed graph, we consider two half-edges of e and orient each half-edge separately. If e is positive, then half-edges must be oriented consistently, that is, if one half-edge is oriented towards the corresponding end vertex of e, the other one is oriented from the corresponding end vertex. If e is negative, both its half-edges must be oriented either towards the corresponding end-vertices of e or towards the centre of e.

A nowhere-zero k-flow of a signed graph G is an orientation of the edges of G together with a mapping $\Phi : E(G) \to \{\pm 1, \pm 2, \ldots, \pm(k-1)\}$ such that at each vertex the sum of incoming values is equal to the sum of outgoing values.

This concept was introduced by Bouchet in 1983, who inter alia conjectured that each signed graph that admits a nowhere-zero flow has a nowhere-zero 6-flow. There are several papers trying to approach this bound for general signed graphs as well as for signed graphs with certain edge-connectivity restrictions. On the other hand, only few results are known for particular classes of graphs. We prove that if a signed complete graph or a signed complete bipartite graph admits a nowhere-zero flow, then it admits a nowhere-zero 4-flow. Moreover, we characterise signed complete and complete bipartite graphs with a nowhere-zero 3-flow and a nowhere-zero 2-flow.

Abelian lifts of complete graphs with loops and multiple edges

David Mesežnikov

Slovak University of Technology, Bratislava

In this contribution we will improve the known upper bound on the Abelian lifts of diameter two and given degree d, which is approximately $0.932d^2 + O(d)$ (published in *J. Šiagiová, A Moore-like bound for graphs of diameter 2 and given degree obtained as Abelian lifts of dipoles, Acta Math. Univ. Comen. 71 (2002) 2, 157-161*). It was obtained by lifting dipoles. In our construction we consider Abelian lifts of complete graphs with the same number of loops on vertices and with the same number of parallel edges between each pair of vertices. If the base graph has, for example, three vertices our upper bound is approximately $0.974d^2 + O(d^{3/2})$.

More symmetries occurring in an infinite word

Edita Pelantová and Štěpán Starosta

Czech Technical University, Prague

Given an infinite word $\mathbf{u}$ over a finite alphabet $\mathcal{A}$, we generalize the notion of palindromic defect of $\mathbf{u}$. Palindromic defect measures the saturation of $\mathbf{u}$ by distinct palindromes. A palindrome is a word invariant under the reversal antimorphism, for instance words $0, 00, 010$ are palindromes since they are read the same from the left as from the right. Words having palindromic defect 0 are fully saturated by palindromes.

The generalization of palindromic defect respects more symmetries occurring in $\mathbf{u}$. These symmetries are given by a finite group G consisting of morphisms and antimorphisms over $\mathcal{A}$ such that the set of factors of $\mathbf{u}$, i.e., the set of all finite contiguous subsequences of $\mathbf{u}$, is invariant under all elements of G. We define the G-defect of $\mathbf{u}$ to be the difference between the maximum number of generalized palindromes (fixed points of involutive antimorphisms of G) and the actual number of generalized palindromes occurring in $\mathbf{u}$.

This notion was first defined using a modification of Rauzy graphs. We also exhibit a class of so-called generalized Thue-Morse words which have G-defect 0, where G is isomorphic to a dihedral group.

On Folkman numbers of graphs and hypergraphs

Reshma Ramadurai

Masaryk University, Brno

Let r be a natural number and G be a graph of order n. It is known that there exists a graph H such that the clique number of H is the same as that of G and every r-coloring of the vertices of H yields a monochromatic and induced subgraph isomorphic to G. The induced Folkman number, denoted by $F(G, r)$, is the minimum order of a graph, H, satisfying the above properties.

This talk pertains to obtaining upper bounds for $F(G, r)$. We are able to quantitatively extend previously known results about $F(G, r)$, by conditioning on the clique number, ω, of G; and show that using our proof technique, this bound is best possible up to a polylogarithmic factor.

I will also talk about some extensions and variations of the classical Folkman problem for hypergraphs.

Complexity of the regular covering problem

Michaela Seifrtová

Charles University, Prague

The concept of graphs covering has been studied since the beginning of the last century. There are many different approaches to it, each of whose shows them in a different light and provides new findings. We concentrated on the complexity of regular covering problem, for which it was useful to construct graph covers via voltage assignments.

It is known, that the H-cover problem can be solvable in polynomial time for some classes of graphs, but Kratochvíl, Pruskurovski, Telle and Fiala have found also a class for which it is NP-complete. However, regularity is a strong condition and it shows up, that the H-regular cover problem, i.e. the question, whether for a given graph H an input graph G does cover it regularly or not, is solvable in polynomial time in many cases. And, unlike the general cover, it is not dependent on the structure of the covered graph, but only on the ratios of the covered and covering graph.

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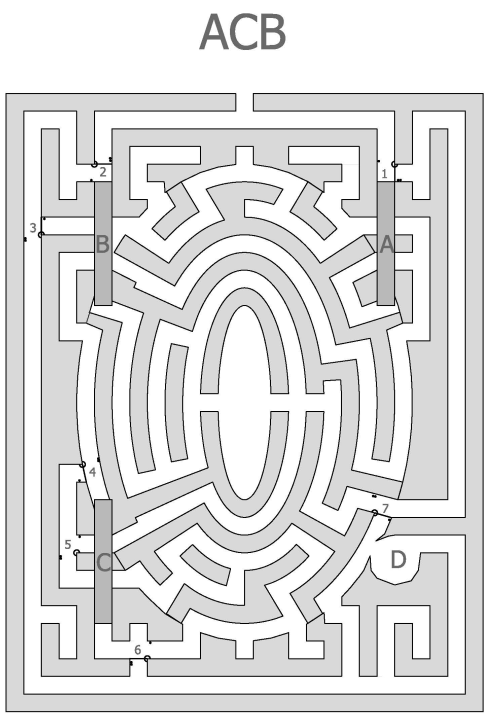
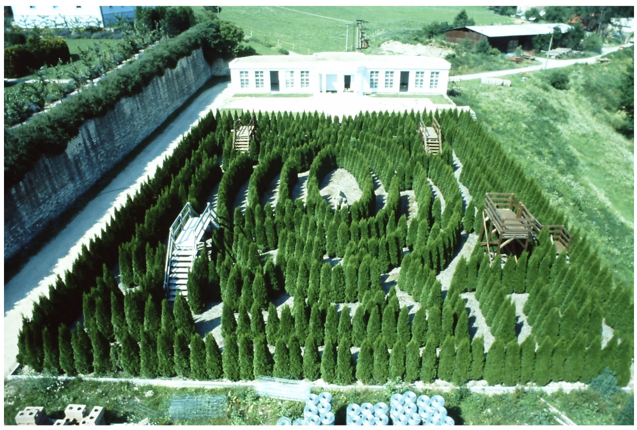
Labyrinth in the Castle of Nové Hrady

The labyrinth is a part of the gardens near the Castle of Nové Hrady. It is already plant out, but is not completely finished yet. According to standard terminology, it is more precise to call it a maze since there is not a single path to the center but the path has to be find out. The full name will be 'Minotaur's labyrinth' reminding the historical Cretan labyrinth hiding this mythical creature.

The aim is to find a path to the viewpoint (the point D) where the whole maze can be observed. There are several dead ends along the way. Currently, the gates that change paths in the maze are not installed yet, so the search is simplified. In future these gates will allow to set up a single path through the maze passing through all the bridges A,B,C in a given order. The possible orders with different paths are ABC, ACB, BAC, and BCA. Below is an example set up for the order ACB.





Beware the monster (of not so distant history) hidden in the maze !