INSIDE OUT, OUTSIDE IN (MATHEMATICS ON A SINGLE SNAPSHOT)

Jaroslav Nešetřil* Miroslav Petříček †

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Mathematisation of sciences seems to be an easy process: one simply isolates basic rules or axioms, one verifies (in some sense) these truths and then all what it remains is to to proceed logically, again following another set of easy rules. However this very schematic approach is far from reality and in full generality, and in precise mathematical terms, this is not even possible. If ever, it is followed in very simple instances only. But many people nevertheless tend to think of mathematics as a collection of trics to solve things.

From another point of view the above approach scratches only the surface of very deep problems. To understand what is true meaning of mathematical "things", which are by nature very abstract, is difficult at *every* level, even the basic one. To quote a few lines from beautiful and moving Khinchin classics:[1] "You must remember that in mathematics (and probably in any other science) the assimilation of anything really valuable and significant involves trying labour."Yes, it is difficult to understand true meanings, role and implications of even of the most elementary notions like set, group, continuity, connectivity. It is difficult to teach these seemingly superficial elementary notions which resist disscours. Only masters can do so, see e.g. enchanting book by Alexander Zvonkin [2].

This dichotomy (one would like to say opposition) of depth and surface is manifested in mathematical world in many instances and it stands behind many everyday aspects of life of most mathematicians.

Consider, for example, the dichotomy between formal and intuitive aspects of mathematical world. The standard style of mathematical writing is lucid, formal and perhaps too concise. This style of writing of most mathematical texts which may be partially responsible for impressions on general public. The recent trend of writing "popular"texts seem to be an indication that the mathematical

*Faculty of Mathematics and Physics, Charles University, Prague

†Faculty of Philosophy, Charles University, Prague

[1] A. Y. Khinchin: Theree pearls of number theory, Gralock Press 1952, Dover 1998.

[2] A. Zvonkin: Mathematics from three to seven, Amer. Math. Soc. 2011 (an English translataion of Russian 2007 original)

community is aware of this problem, see e.g. Tim Gowers [3]. (Also this article is not a pure mathematical text.) On the other hand the mathematical intuitions (not to speak about emotions) are usually hidden as if they did not matter. But all working mathematicians know how important they are and what pivotal role they play. These opposing aspects are demonstrated also in two main lines of mathematical activity: theory building and problem solving. This was recently intensively discused [4] in the framework of another phenomena of *two cultures* (see also Introduction to [5]).

Why do we reveal this here? Why are these peculiarities of mathematical praxis important? The point we want to make is exactly this: there is nothing particular about mathematics here and that any deep thinking seem to have similar aspects. Similarity consisting not in the contents but in the form, in the way how things are being organised and revealed. We want to illustrate this by means of a very concrete example. A single snapshot will be enough for us. This particular method, i.e. to demonstrate a complicated situation by means of a simple example, is very close to mathematical thinking and seems to be appropriate here. Also in the art history it has been also applied, see e.g. [6] and a beautiful book by T. J. Clark. [7]

What is then the leitmotiv of this article? We mentioned above *depth* and *surface* and this text is about it.

The opposition of depth and surface seems to be a quite unquestionable one, not least from the psychological point of view. Philosophy, too, especially the phenomenological tradition which constitutes one of the basic sources for contemporary humanities, considers this opposition as fundamental for human orientation in a lived world insofar as the notion of the horizon (i.e. the depth dimension of all our perceptions guiding all acts of intentionality) functions in it as an *apriori* condition of human understanding.

But when this opposition occurs in the metaphorical sense it is not so unquestionable, because in such cases we can find many examples denying the evidence of the opposition. In many cases this is due to the fact that cultural evaluation of it runs contrary to the spatial basis, or vehicle, of the metaphor. For example in biology the depth-surface opposition seems to be valid in an inverted way. This was shown by Adolf Portmann and Hannah Arendt: living beings live for the sake of their appearing, that is for the sake of the outer appearance, the surface, and not vice versa; therefore "the relevant and the meaningful in this world of ours should be located precisely on the surface."[8] To this fundamental orientation of life corresponds the fact that in the case of living beings their outward appearance is almost infinitely diverse and differentiated, that it fulfils the role of individuation, whereas "the inner, non-appearing, organs exist only in

[3] T. Gowers: Mathematics - a very short introductions, Orford University Press 2002.

[4] T. Growers: Two cultures in mathematics. In Mathematics: frontiers and perspectives (Amer. Math. Soc., Providence, RI, 2000), 65-78.

[5] R. L. Graham, J. Nešetřil, S. Butler: Mathematics of P. Erdős, Springer 2013

[6] S. Lücke-David: Max Ernst "Euclid", Books on Demand GmbH, Norderstedt, 2008

[7] T. J. Clark: The Sight of Death An experiment in Art writing, Yale University Press, 2006.

[8] H. Arendt, The Life of Spirit. Vol. I. The Thinking, Harcourt, 1978, p. 27.

order to bring forth and maintain the appearances"[9] so that all internal organs look much the same.

An analogical ambiguity, however, can be found in much more exact sciences like structuralist linguistics or structural anthropology. According to Lévi-Strauss, the structure should be unconscious (therefore deeply hidden under the surface), but detailed examination of communication, or collective human behaviour, shows that it is impossible to strictly separate the level of surface and the level of depth, that there is rather a relation of reciprocal determination between these two levels: the structure (depth) determines the structured (surface).

This reciprocity can be seen clearly also in mathematics. In mathematical terms every good problem leads to a theory and every good theory leads to a interesting new definitions, in fact to a space of notions which in turn provoke new problems. One behaves responsibly and one defines only what is needed to be defined. The elegance is a profound criterium. Metaphorically this has been codified by the Paul Erdős story of *The Book*: There exists a book, The Book, which is in the possession of God which contains for each (true) mathematical statement a most beautiful short proof so called Book Proof. We all try to imitate the book proofs and there is even a book attempt for it [10]. In more philosophical terms, this cognate reciprocity problem is faced by Edmund Husserl's phenomenological theory of language for which the expression as the outward form of meaning and as the manifest level of the sense which is "intended"should be no more than an instrument, and therefore unproductive – it should not in any substantial way participate in the constitution of meaning. However, this idea is ultimately untenable: the way, in which something is expressed is involved in the creating of sense, the (communicable) meaning must be made external in order to be general, and general in order to be communicable. Again, surface and depth are interdependent.

It might be noted quite generally that in the culture of the 20th century the relationship between depth and surface has been gradually reassessed, as is evident for instance in the well-known proclamation of Paul Valéry "devenons superficiel"from his essay Le problème des musées (1923) or in Sigmund Freud's concept of the unconsciousness as a mere surface and, last but not least, in Maurice Merleau-Ponty's attempt to grasp the relation of the external and the internal in terms of "chiasmus, that is an intertwining". The late philosophy of Merleau-Ponty as a whole is based not on the opposition but on the reversibility of the "flesh of the world"and the "flesh of the body"as experienced through the striking affinity of the act of perceiving and the perceived objects (an incarnate eye sees corporeal objects). In such a manner Merleau-Ponty breaks down the boundary between subjective and objective reality, but he goes a step further and finally discovers the extraordinary harmony of the external and internal, which "is possible only through the mediation of a positive infinite or (since every restriction on a certain kind of infinity would be the seat of negation) an

[9] Ibid., p.28

[10] M. Aigner, G. Ziegler: Proofs from The Book, Springer 1998

infinite infinite. It is in this positive infinite that the actual existence of things partes extra partes and extension as we think of it (which on the contrary is continuous and infinite) communicate or are joined together."[11]

It might certainly be argued that this confusion in distinguishing surface and depth, which seems to reflect both the famous maxim of Johann W. Goethe "One should not see anything further behind the phenomena: they themselves are the theory"[12] and Nietzsche's description of a world without God, in which man finally raises the questions "What were we doing when we unchained this earth from its sun? Whither is it moving now? Whither are we moving? Away from all suns? Are we not plunging continually? Backward, sideward, forward, in all directions? Is there still any up or down?"[13], is simply due to the metaphorical use of relations pertaining only to physical space, so that this ambiguity might be seen as a consequence of inscribing these relations into certain cultural frames.

On the other hand, it would be possible to counter this objection – somewhat paradoxically – by reminding ourselves that the deep modification of the depth-surface relation is at work in the basic philosophical conceptions which slowly became the main frame of reference in the thinking of 20th century and which – supported by bold thought-experiments – rose up against the (not always fully justified) assumptions of traditional Western philosophy and culture. And, secondly, it could be argued, too, that these thought-experiments were preceded by very real revolutions in art (and science), as attested – among many other things – precisely by the interest shown by avant-garde artists in the new possibilities of visualisation, which ocured almost simultaneously with the invention of photography. But this state of affairs is not so paradoxical, as it could seem at first glance, considering that photography was, due to its exactness and objectivity, for many avant-garde artists the means that enabled them to discover other ways of representing reality, to examine hitherto unnoticed aspects of it, especially as this new technical instrument of discovery was largely independent of the anthropomorphic or subjective conditions of perception – the automatic registration of observed facts is very close to the ideal of scientific inspection. The use of the photographic apparatus linked together art and science: experimentation with photography was a sort of artistic creation as well as laboratory research. What is at stake is not only *Neue Sachlichkeit* the objective style of the precise photography, but *Neues Sehen* which emphasises the unproven ways of using the camera and discovers in this manner new possibilities of representing space and/or of conceiving it by means of these other ways of representing it. The technology here is neither merely an extension nor a replica of the human eye, it is not a prolongation of human culture, which was for centuries associated with representations of the world based on the perspective of (traditional) painting, but the technology incorporated in the

[11] M. Marleau-Ponty, Signs, trans. Richard McCleary, Evanston, Northwestern University Press, 1964, pp. 148-149.(Signes, Gallimard, Paris 1960,p.187).

[12] J. W. Goethe, Scientific Studies, ed. and trans. Douglas Miller, vol. 12 Collected Works in English, New York, Suhrkamp, 1988,p.307.

[13] F. Nietzsche, The Gay Science (1882 1887), § 125, Walter Kaufmann ed., New York: Vintage, 1974, pp.181

camera becomes, literally, a *machine of vision* producing de-centered images without recognisable axis, obscuring the bottom and top, because it takes its pictures from unexpected, "unnatural"angles revealing thus a wholly new visibility. Perhaps the best example of this new visual search is the well-known photographical cycle *Equivalents* of Alfred Stieglitz, who quite deliberately creates images of space without a clear and distinct horizon, producing the photos as "small fragments of infinity". The first photomicrographs, we might note by the way, were taken in the 1850s and 1860s (Fox Talbot and Adolphe Bertsch), and they were soon followed by aerial pictures, celestial photography and X-ray photography.

It is advisable to have all these facts in mind when looking at pictures from that time, and even at those photos that seem to be pretty much conventional – for example, when looking at László Moholy-Nagy's photograph of 1928 which bears the title *Radio Tower Berlin* and which, indeed, deserves special and attentive treatment.

It would be certainly possible to say that Moholy-Nagy is attempting to take this picture from an unusual angle. That was, in the 1920's, in no way unusual. Similarly, André Kertész, his compatriot, photographed the Eiffel Tower, and several others were fascinated by strange views of the cast-iron constructions, such as Germaine Krull, whose photos of the Eiffel Tower were taken around the same year. Indeed, she used this principle of the unusual point of view directed, as it were, from outside towards the inside quite frequently (see, for example, her shot of the *Théatre Pigalle* of 1929). In all these pictures there is a remarkable tension between the documentary nature of the photography, its objectivity, and modern technology which calls for a different aesthetic, as advocated in these years mainly by constructivists. But, obviously, Moholy-Nagy does not seek something like the effect of "defamiliarisation", he does not seek a different way of photography – all his photos are an integral part of his artistic creation as a whole. He was a visionary who in the years 1923-1928 lectured at *Bauhaus* where he had among his colleagues Paul Klee and Wassily Kandinsky. He was engaged in analytical research into the new technologies of reproduction, testing them and identifying their possibilities; in addition to photos he created movies, collages and photomontages and so called photograms, i.e. photographs created by allowing the light to pass through various transparent or semi-transparent objects and to be fixed directly on the photosensitive layer of photographic paper. But he also painted images and made kinetic sculptures. His numerous texts from this period show that he was mostly interested in the relationship of light and space (for example, he wrote: "Fotografie ist Lichtgestaltung") and in the camera's ability to construct space in a different way to the space "constructed"by the human eye. He was convinced that the possibilities of photography exceed the mere mechanical registering of reality and that the very mechanical aspects of the camera necessarily modify human relationship to space. [14]

[14]L. Moholy-Nagy, "Espace-temps et photographie". In: Peinture. Photographie. Film et autres ectrits sur la photographie. Gallimard 2007, p. 234. ("Space-Time and the Photogra-

That is why photography is for Moholy-Nagy an essential "tool".In the short text *A New Instrument of Vision* he writes: "In photography we are in possession of an extraordinary instrument for reproduction. But photography is much more than that. Today it is in a fair way to bring (optically) something entirely new into the world. The specific elements of photography can be isolated from their attendant complications, not only theoretically, but tangibly, and in their manifest reality."[15] Sure, it can reproduce or represent, but this is not its only, or problem-free, ability, for if we overestimate just this ability we come back to the issue of conventions (two-dimensionality, black and white image etc.). What affords the photograph its unique position among images is the fact that it is simultaneously an iconic and an indexical "sign", as was already noticed by Charles Sanders Peirce, one of the founders of modern semiotics. While showing a resemblance with the photographed object it also refers to the fact that this object really existed, that it stood before the camera lens, so that the photo – as its imprint fixed in light-sensitive material – is the trace of this object; and a trace, of course, is an indexical sign.

Photography testifies to the existence or reality of the photographed thing. Therefore the distinction between artistic and non-artistic use of photography becomes temporarily (in the 1920s and 1930s), as well as locally (in the milieu of the avant-garde artists), rather fuzzy, and the act of photographing becomes a unique field of experimentation, that is the field of the search for the unseen in the seen, or for that which Walter Benjamin, in his essay "The work of art in the age of its technical reproducibility"called the *optical unconsciousness* [16]. A photographic apparatus can make things visible which are impossible to grasp by means of the naked eye, as was very early demonstrated by E. Marey and E. Muybridge. The notion of the "unconscious"borrowed from Freud means the following: there is a reality which is not beneath the realm limited by human physiology, but which inheres in the folds of the common visual field.

Once again, Moholy-Nagy does not seek to denaturalise or "defamiliarise"– the effect of estrangement is only a way of visualising a certain concept. Technologies make it possible to reveal other *articulations* of space. In the wonderful age of *pilotable* airships and airplanes we can look at reality from above and "these new possibilities add another extraordinary, almost indescribable, perspective to our lives,"as he wrote in the article "Space-Time and the Photographer"in 1943. [17] In brief, space is nothing given, it has its history during which (considering the different strategies of representation from ancient Egypt to the Greek Pantheon and further) it slowly reveals its dimensions; in this way – and not least through photographs – there arises in our time "a new language in the domain of the space-orientation and communication." [18] Technology (photogra-

pher", American Annual of Photography, Boston and London, no. 57, 1943.)

[15]In: L. Wells (ed.), The Photography Reader, Routledge, London and New York, 2003, p. 92.

[16]W. Benjamin, "Kleine Geschichte der Photographie", *Gesammelte Werke*, Suhrkamp, Frankfurt am Main, Bd.II, p. 371

[17]L. Moholy-Nagy, *Peinture, Photographie. Film et autres écritssur la photography.* Gallimard 2007, p. 237

[18]Ibid., p. 238

phy, film) outruns our so called natural perception, and its experimental-artistic use leads us to revise the traditional concepts of space, including the relationship of the surface and depth, or quite generally, metric and non-metric space. So it becomes possible to explain why photographic experiments stand alongside experiments with non-figurative painting. All these efforts are in fact serious *investigations*: the transformations of the real objects or forms pose a challenge to our understanding of space as well.

A few comments are in order here: Any new experiment in life, science and art calls for a proper understanding and calls for investigations. And any true novelty needs great courage. The motivation may be obscure (and even not true; an example for many: think about motivations for Columbus journey), but it gives to our endeavour depth and energy. Abstract art is a good example when some of the key artists proceeded in their investigations in the seemingly unrelated context (e.g. Kupka – occultism, Kandinsky – music). In the same way mathematicians (and computer scientists) like to visualise things which are strictly speaking impossible to visualise. In a rather romantic way the mathematical abstract notions may get anthropomorphic forms and even numbers may get colours.

In Moholy-Nagy's photograph of the *Radio Tower Berlin* all these features are quite apparent: there is the dramatic, rapid-diagonal, high-angle, view, using a lens with long focal length that reduces the depth of the visual field so that everything seems to be situated on the same plane; all the objects in this picture enter into different relationships of proximity and distance than those of "natural"space. The result is an image that is close to being some peculiar form of diagram. Certainly, each photographic camera produces a two-dimensional image of three-dimensional space, but this disappearance of depth may always be suppressed by the use of a long focal lens. Moholy-Nagy, however, avoids such tricks quite deliberately:"The camera has offered us amazing possibilities, which we are only just beginning to exploit. The visual image has been expanded and even the modern lens is no longer tied to the narrow limits of our eye... nor should we regard the ability of the lens to distort - the view from below, from above, the oblique view - as in any sense merely negative, for it provides an impartial approach, such as our eyes, tied as they are to the laws of association, do not give."[19]

Any new mathematical tool leads almost immediately to investigations on its own: It puts the previously known facts in new (perhaps more natural and easier) context, it may answer something which was not known before, it may contribute to understanding of complexity of our knowledge which is of course temporal. And we ask all the time typical questions involving concepts like dimension, algorithm, decomposition or universality and representation. It is amazing that this can be almost verbatim traced in our example in a very different area of (a newly created area of) art. Thus we restrict our comments on

[19]L. Moholy-Nagy, *Painting, Photography, Film.* With a note by Hans M. Wingler and a Postscript by Otto Stelzer. Translated by Janet Seligman. MIT Press, Cambridge, Mass. 1967. Original German edition: *Malerei, Fotografie, Film*, Bauhausbücher Vol. 8, 1927 (2. ed.), p. 7.

science in favour of comments tracing Moholy Nagy' s art. From the point of view of "lived space"and with regard to Moholy-Nagy's experiments, we might speak of a bifurcation of the meaning of "place": as opposed to the sense of "emplacement"(which is associated with human ways of occupying space and which means a specific "locality", which is a place *qua* basic human manner of being in the world, or an "existential"according to Martin Heidegger) there is the sense "site"(a rather general spatial location or a general position in general space), but – in the case of photography of the *Radio Tower Berlin* – it cannot be maintained that one meaning dominates over the other, regardless of the fact that objects and people on the ground are changed into practically geometrical points, because at the same time we perceive the protruding movement of the tower, experiencing it almost as a living organism. Similarly, one can examine in this picture the strange relationship of organic and non-organic forms: here they are not side by side, but seem to penetrate one into another and are nearly interchangeable: the strict geometric construction of the tower reminds one of organic growth, so that it passes on the side of the "intensive"or non-metric space. Of course this is meant intuitively but it is interesting to note that this period of (20ies and 30ies of last century) was the time of rapid development of new mathematical models of space and geometry. This was the time when "non-metric"models such as topology and then algebraic (at that time called combinatorial) topology were developed and were developing very quickly. It needed a great courage and intensive works of some of the great mathematicians of that time to describe space just by neighbourhoods, or coverings and/or abstract convergence only. This was a prime mathematical activity which influenced a larger part of abstract mathematics of XX. century, see for example [20].

While the tower is treated in such rather unusual and complex space, the restaurant, that is the human world, in the vicinity of the tower at its foot, shows off its exact ordering and passes off as "extensive"or a metric (in the sense of usual) space. Both aspects blend together because the one is contained in the other, and it is impossible to draw a clear line of demarcation between the two. That is why this photo, though it presents static objects, is extremely dynamic. However: it is a photograph – not a fiction or artificial construction; therefore it is impossible not to believe it, because, as already mentioned, photography is not only iconic, but also an indexical "sign"; it is thoroughly objective, for it says: "This is the way things are."

And yes, this objectivity and "merciless truth"is the essence of mathematics. You may not like it, or you may not understand it but the mathematical findings are beyond any doubts and discussion true. Mathematics does not just mirror the reality, it abstracts, modifies and generalises it but what it finds and claims is true (although the truth may be sometimes inconvenient). And from the very beginnings of his artistic career László Moholy-Nagy put a strong emphasis on precisely this aspect of photography: "... the photographic camera

[20] H. Whitney, *Moscow 1935: topology moving toward America*, In: A century of mathematics in America, Part I, Hist. Math., vol. 1, Amer. Math. Soc., Providence, RI, 1988, pp. 97–117.

reproduces the purely optical image and therefore shows the optically true distortions, deformations, foreshortening etc., whereas the eye together with our intellectual experience, supplements perceived optical phenomena by means of association, and formally and spatially creates a *conceptual image*. Thus, in the photographic camera we have the most reliable aid to a beginning of objective vision. . . "[21]

The composition of the photo of the *Radio Tower Berlin* is undoubtedly highly sophisticated and well-thought-out, something which is apparent from the fact that the picture fits very well in the overall context of his work as a whole. Its diagrammatical disposition, for example, clearly refers to the search for a universal visual language; in the 1920s this search brought together all artists engaged in photography and film (Hans Richter, Vigg Eggeling, several members of Bauhaus as well as Dziga Vertov and others). What is really important is not reproduction, but the specific organisation or orchestration of forms and the effort to grasp, by means of the camera, the rhythm of objects in space and time. At such a level of "abstraction"it is possible to achieve a "universal validity"comparable to the exactness of the language of mathematics which Moholy-Nagy claimed for photography. Photographic presentation of reality goes beyond the usual conventions of seeing – it is something like "typography", proper to reality itself – it is visual speech. And Moholy-Nagy also experimented with a combination of letters and photographs which he called "typophoto". He probably went furthest in this direction with his photograms. As is clear from his posthumously published book *Vision in Motion*, this search for the universal language of reality is also closely associated with the exploration of space. Even in his films he deals with the blending of the organic and non-organic forms, or metric and non-metric space. For example, his documentary *Impressionen vom alten Marseiller Hafen (Vieux Port)* from the year 1929 begins with a shot of a map (metric space) which is slowly torn apart revealing a busy intersection full of life (giving an illusion of a non-metric space) filmed from above. One of the visually most impressive images in this film is a shot from the height of the giant suspension-bridge looking into its construction and down to its anchoring and to its surroundings. Although it is a "figurative image", it gradually moves towards the avant-garde's films of this time which tried to produce a sort of abstract "moving images": that is, movement in space based on the direct evolution of pure forms. In this regard, too, Moholy-Nagys picture of the bridge in Marseilles is very close to the photo of *Radio Tower Berlin*: its aim is not to represent, but to investigate another possible articulation of space itself. In the essay entitled "A new instrument of Vision", which deals with photography, Moholy-Nagy wrote: "Through photography, too, we can participate in new experiences of space, and in even greater measure through film. With their help, and that of the new school of architects, we have attained an enlargement and sublimation of our appreciation of space, the comprehension of

[21] L. Moholy-Nagy, *Painting, Photography, Film.* With a note by Hans M. Wingler and a Postscript by Otto Stelzer. Translated by Janet Seligman. MIT Press, Cambridge, Mass. 1967. Original German edition: *Malerei, Fotografie, Film*, Bauhausbücher Vol. 8, 1927 (2. ed.), p.28 .

a new spatial culture. Thanks to the photographer, humanity has acquired the power of perceiving its surroundings, and its very existence, with new eyes."[22] The photo entitled *Radio Tower Berlin* might, then, be seen as a sort of introduction into this new "spatial culture". Its paradox lies, *inter alia*, in the fact that the tower is visible as if from the inside and outside simultaneously (thus, the image does not try to suggest three-dimensional space, but at the same time it makes doubtful its own two-dimensionality). As if the perception of space was visual as well as tactile insofar as touch is a sense which does not distinguish the opposition of outside and inside. Moreover, touching is always a sensation of surface, but the word "surface"has a peculiar meaning in this case since it is not defined as the antithesis of depth. Perhaps therein lies the aforementioned "sublimation" of the human relationship to space; it might be that photograms are another form of this sublimation: when describing a similar process of creating images to the one practiced by Man Ray, Moholy-Nagy says of his "rayographs": "the laws of gravity seem to be wholly abolished, the objects are floating in a magical space". [23] In a "magical"space, because its dimensionality is unclear when seen in terms of the three-dimensional or two-dimensional space only.

But it is important to note that Moholy-Nagy's experiments with space and light in photography, film or painting and sculpture are always situated in the framework of an endeavour to attain absolute objectivity both as concerns his methods and as concerns the final artistic results. Therefore not only photography, but also the technology of photo-production, is of paramount importance; the "photographical"manner of seeing is not affected either by the traditional conventions of representing reality (as mediated especially by the tradition of painting) or by specific properties of the human form of vision. Photography is "one of the most important factors in the dawn of a new life"because it allows one to achieve "mathematical certainty".[24] The space re-presented in the *Radio Tower Berlin* is not deformed, but it is actually "sublimated"in the process of "elucidation and purification". The strange blending of the organic and non-organic forms, of surface and depth or of top and bottom, in short of metric and (intuitive) non-metric space, points to what might possibly be the culmination of such sublimation: *the space which is proper to light*, thus somehow absolute, or absolutely pure, space: *the space of all spaces*. However, photography as an image, that is as a specific sign, is not only an index, but also an icon, so that in some way it can *represent* this space as the reality which undoubtedly obtained before the lens of the camera at the moment that this photo was made – and at the same time it assumes almost an "abstract"complexion. In this ambiguity it would be possible to look for an answer to the question of why Moholy-Nagy claimed the photogram as the most completely dematerialised medium which the New Vision commands. And perhaps it could be argued that this successive process of sublimation includes his paintings as well as his sculptures, especially the most famous of his artefacts, namely the sculpture *Licht-Raum Modulator*.

[22] *Peinture. Photographie. Film.* p. 216. (Telezor, Brno, Nos. 1-2, 1936).

[23] *Peinture. Photographie. Film,* "La Réclame photoplastique"("Photoplastische Reklame", Offset, Buch ind Werbekunst, 1926), p. 142.

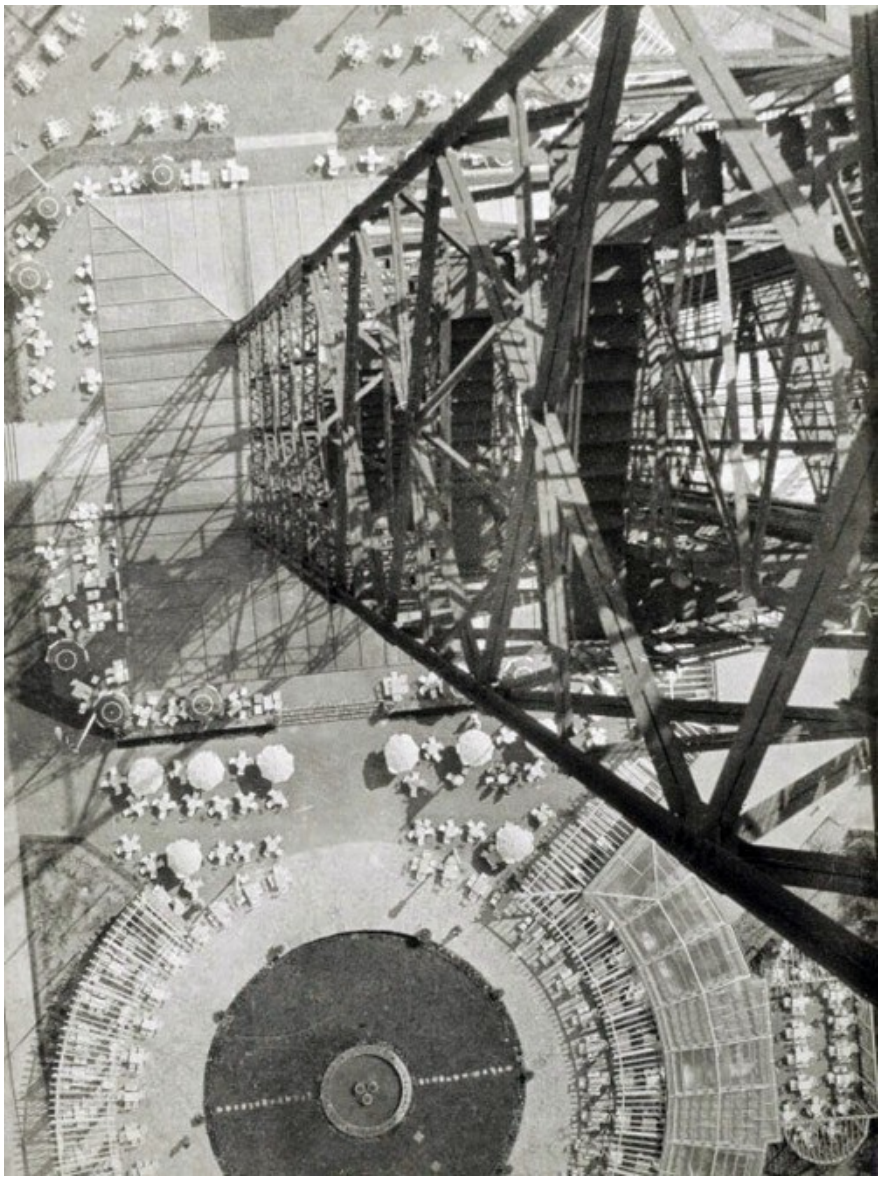
[24] L. Moholy-Nagy, Painting. Photography. Film, p. 27."

In this sense it would be possible to speak of an abstract space – and even to go a step further and to distinguish two basic paths to abstraction in art, as it is born at this time. László Moholy-Nagy would represent that kind of abstraction which arises out of the sublimation of space – and not (as the other path) through the distillation of tonal, musical order, through the purification of rhythms (Arnold Schönberg – František Kupka and Wassily Kandinsky). The gradual formation of the first type of abstraction resulting from the sublimation of space could then be traced in several abstract works of Moholy-Nagy. For example, his untitled photogram from 1941 or the final illustration in his book *The New Vision* (1938 edition) are reminiscent of the photo of the *Radio Tower Berlin*, as if it went through another and higher phase of sublimation: though they should be "flat", they evoke a specific depth. But this line of development to higher space-abstraction also runs from the photo of *Radio Tower Berlin* to Moholy-Nagy's compositions designed only by letter and numbers (CH X, 1939) or to the somewhat enigmatic Study with Pins and Ribbons (1937-38). All these works destabilise the place of the spectator: though they are "flat"images, they nevertheless cause dizziness and vertigo (in the same way as his photos from the 1920s, *Dessau, Ascona or Rothenburg*): the iconic presentation of the space of spaces to which the photograph of the *Radio Tower Berlin* points in an indexical way.

Art thus rehabilitates the process of mathematisation, which Edmund Husserl situated at the beginning of the modern era and which he regarded as the start of the true sense or *telos* of science, as it was born in Europe. But what is at stake, now as in the 20ies, is the other mathematisation which concerns not the objects of perception, but the space itself where these objects can be perceived. And experiments of classical moderne implied that the notion of Euclidean space has nothing to do with so called "natural perception", no matter how unproblematically this notion was for centuries regarded as the essential element of such perception. This motivations continues, in a different context, until now.

And again it is not a coincidence that one of the key problems of mathematics and both theoretical and engineering computer science is to find proper models and provide understanding of complex geometry of networks which constitute us humans and in which we all live. Very large dynamic networks are actual reality (and not an asymptotic or limiting abstraction) and their study transcendence particular disciplines (biology, physics, computer science, mathematics and social studies).

Sometimes we interpret particular mathematical findings (or scientific imaginary) in aesthetic context. But it is very rare that we have an opportunity to illustrate scientific thinking by means of artistic processes. That may sound even as an obscure idea or simply impossible. In this paper we tried to give such an example. Maybe this short text could not convince our reader but perhaps some affinity between thoughts of 20th century modernists and (eternal) strategies of mathematicians became evident.



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