Algebraic, Topological and Complexity Aspects of Graph Covers

&

Winter School in Harmonic Functions on Graphs and Combinatorial Designs

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Welcome

to the workshop on Algebraic, Topological and Complexity Aspects of Graph Covers & Winter School in Harmonic Functions on Graphs and Combinatorial Designs.

This is the sixth workshop in the ATCAGC series, whose previous locations were sometimes intentionally chosen in remote places far from civilization: Bovec (Slovenia, 2013), Eugene and Breitenbush (Oregon, USA, 2012), Král'ova Studňa (Slovakia, 2011), Auckland (New Zealand, 2010) and Finse (Norway, 2009). We have followed this idea and chosen hotel Sepetná in Beskydy mountains, almost on the border between Czech Republic and Slovakia.

We believe that you will enjoy the programme constituted from two invited lectures and 11 contributed talks. The invited lectures will be given by Marston Conder from the University of Auckland who will speak on "Rotary and regular maps with simple underlying graphs"and by Jozef Širáň from Slovak University of Technology in Bratislava who will present a lecture entitled "Covering constructions of extremal graphs of given degree and diameter, or girth". The winter school consists of two intensive courses: "Harmonic covers of graphs"given by Alexander Mednykh from the Novosibirsk State University and "Combinatorial Designs"given by Alexander Rosa from the McMaster University.

The workshop is organized as a joint venture of Czech Mathematical Society, section of The Union of Czech Mathematicians and Physicists, Slovak Mathematical Society, section of The Union of Slovak Mathematicians and Physicists, Matej Bel University, Banská Bystrica, and Department of Applied Mathematics, Charles University, Prague. We also gratefully acknowledge sponsorship of the workshop and the winter school by the grant mobility project: "Mobility — Enhancing Research, Science and Education"ITMS code: 26110230082, under the Operational Programme Education cofinanced by the European Social Fund, and by Czech research grant GAČR P202/12/G061: Centrum excelence — Institut teoretické informatiky (CE-ITI).

We wish You a pleasant and inspiring stay at ATCAGC 2014.

Jiří Fiala
Jan Kratochvíl
Roman Nedela
Michaela Seifrtová

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Part I

ATCAGC Invited Talks

Rotary and regular maps with simple underlying graphs

Marston Conder, Jicheng Ma

University of Auckland

A regular map is a symmetric embedding of a graph (or multigraph) on some closed surface. This talk concerns the genus spectrum for such maps on orientable surfaces, with SIMPLE underlying graph. It is known that for some positive integers g, there is no orientably-regular map of genus g for which both the map and its dual have simple underlying graph, and also that for some g, there is no such map (with simple underlying graph) that is reflexible. I will show how families of covers of graphs/maps can be used to prove that that for over 83% of all positive integers g, there exists at least one orientably-regular map of genus g with simple underlying graph. This is joint work with my former PhD student Jicheng Ma. We conjecture that there exists at least one such map for every positive integer g.

Covering constructions of extremal graphs of given degree and diameter, or girth

Jozef Širáň

Open University
Slovak University of Technology

The degree-diameter problem is to determine the largest order of a graph of a given maximum degree and diameter. The related problem of determination of the smallest order of a graph of a given degree and girth is sometimes called the degree-girth problem. In both cases our knowledge about the actual maxima/minima is rather limited. Nevertheless, there are a number of interesting bounds on these parameters obtained by constructions.

It turns out that a considerable proportion of the largest currently known graphs of given degree and diameter, and the smallest known graphs of given degree and girth, have been or can be obtained as lifts of small base graphs with voltages in groups with a fairly simple structure. In the talk we will give a survey of these constructions and outline open problems in this area of research.

Part II

Winter School Lectures

Graph Coverings and Harmonic Maps in Exercises

Alexander Mednykh

Sobolev Institute of Mathematics

The aim of lectures is to give a background for the theory of graph coverings and harmonic maps.

The basic definitions and main results of the theory are followed by exercises. Some of these excises are elementary, some of them will require non-trivial effort and some of them are unsolved problems. Also, the basic theory will be provided by numerous examples and the most exercises by solutions.

Combinatorial Design Theory

Alexander Rosa

McMaster University

Combinatorial design theory traces its origins to statistical theory of experimental design but also to recreational mathematics of the 19th century and to geometry. In the past forty years combinatorial design theory has developed into a vibrant independent branch of combinatorics with its own aims, methods and problems. It has found substantial applications in other branches of combinatorics, in graph theory, coding theory, theoretical computer science, statistics, and algebra, among others.

The objective of this short course is to convey a solid basic knowledge of problems and methods of combinatorial design theory, up to a level that will enable one to approach open research problems.

Part III

Regular Contributions

Locally constrained homomorphisms on graphs of bounded degree and bounded treewidth

Jiří Fiala, S. Chaplick, P. van't Hof, D. Paulusma and M. Tesař

Charles University

A homomorphism from a graph G to a graph H is locally bijective, surjective, or injective if its restriction to the neighborhood of every vertex of G is bijective, surjective, or injective, respectively. We prove that the problems of testing whether a given graph G allows a homomorphism to a given graph H that is locally bijective, surjective, or injective, respectively, are NP-complete, even in the restricted case where G has pathwidth 5, 4 or 2, respectively, or when both G and H have maximum degree 3. We complement these hardness results by showing that the three problems are polynomial-time solvable if G has bounded treewidth and in addition G or H has bounded maximum degree.

Computing the Stretch of an Embedded Graph

Petr Hliněný

FI MU Brno

The stretch of a graph embedded on a surface is the minimum of $len(\alpha) \cdot len(\beta)$ over all pairs of cycles α and β that cross exactly once. We provide two algorithms to compute the stretch of an embedded graph, each based on a different principle.

Almost totally branched coverings between regular hypermaps

Kan Hu

Matej Bel University

The notion of almost totally branched coverings arose natually from the investigation of regular cyclic (map) coverings of the platonic maps, possibly branched simultaneously over the vertices and over the faces. Such coverings are closly related to maps with multiple edges. The introduction of almost totally branched coverings has enabled us to give a complete classification of cyclic regular coverings over the platonic maps. In this talk, we will consider a generalization of such a notion to the coverings between regular hypermaps. We give a complete classification of almost totally branched hypermap coverings of the platonic maps, possibly branched at the vertices and at the edges, but smooth at the faces.

Algorithmic Aspects of Regular Graph Covers

Jiří Fiala, Pavel Klavík, Jan Kratochvíl, Roman Nedela

Charles University in Prague

A graph G *covers* a graph H if there exists a locally bijective homomorphism from G to H, roughly meaning that G looks locally the same as H. We deal with *regular covers* in which this locally bijective homomorphism is prescribed by an action of a subgroup of $\mathrm{Aut}(G)$. Regular covers have many applications in constructions and studies of big objects all over mathematics and computer science.

We study *computational aspects* of regular covers that have not been addressed before. The decision problem RegularCover asks for two given graphs G and H whether G regularly covers H. When $|H| = 1$, this problem becomes Cayley graph recognition for which the complexity is still unresolved. Another special case arises for $|G| = |H|$ when it becomes the graph isomorphism problem. Therefore, we restrict ourself to graph classes with polynomially solvable graph isomorphism.

Inspired by Negami, we apply the structural results used by Babai in the 1970's to study automorphism groups. Our main result is the following FPT meta-algorithm: Let $\mathcal{C}$ be a class of graphs such that the structure of automorphism groups of 3-connected graphs in $\mathcal{C}$ is simple. Then we can solve RegularCover for $\mathcal{C}$-inputs G in time $\mathcal{O}^*(2^{e(H)/2})$ where $e(H)$ denotes the number of the edges of H. As one example of $\mathcal{C}$, this meta-algorithm applies to planar graphs. In comparison, testing general graph covers is known to be NP-complete for planar inputs G even for a small fixed graph H like K_4 or K_5.

Domination in vertex-transitive cubic graphs on 2^m vertices

Martin Knor, Primož Potočnik

Slovak University of Technology in Bratislava

A vertex of a graph dominates itself and all its neighbours. A set D, subset of vertices of a graph G, dominates the graph G efficiently if every vertex of G is dominated by exactly one vertex of D. Finally, Mobius ladder M_n is a cubic graph obtained from the cycle on $2n$ vertices by adding a perfect matching connecting pairs of opposite vertices. Using an algebraic approach based on lifts and coverings we prove that, a connected vertex-transitive cubic graph G on 2^m vertices does not admit efficient dominating set if and only if $m \geq 3$ and G is isomorphic to the Mobius ladder $M_{2^{m-1}}$.

On Split Liftings with Sectional Complements

Aleksander Malnič, Rok Požar

University of Ljubljana and University of Primorska

Let $\wp\colon \tilde{X} \to X$ be a regular covering projection of connected graphs, and let $\mathrm{CT}_\wp$ denote the group of covering transformations. The problem whether a given group of automorphisms $\tilde{G} \leq \mathrm{Aut}(\tilde{X})$ lifts along $\wp$ as a split extension of $\mathrm{CT}_\wp$ by G such that there exists a complement to $\mathrm{CT}_\wp$ with an invariant section is analyzed in detail. The analysis is done using the encoded information on the base graph X via Cayley voltages without actual inspection of the covering graph or the lifted group.

On Wiman's theorem for graphs

Alexander Mednykh

Sobolev Institute of Mathematics

The aim of this report is to find a discrete version of the Wiman theorem which states that the maximum order of an automorphism of a Riemann surface of genus $g \geq 2$ is $4g+2$. The role of a Riemann surface in this paper plays a finite connected graph. The genus of a graph is defined as the rank of its homology group. Let Z_N be a cyclic group acting freely on the set of directed edges of a graph X of genus $g \geq 2$. We prove that $N \leq 2g+2$. The upper bound $N = 2g+2$ is attained for any even g. In this case, the signature of orbifold $X = Z_N$ is $(0; 2, g+1)$, that is X/Z_N is a tree with two branch points of order 2 and $g+1$ respectively.

On isospectrality of genus two graph

Ilya Mednykh

Sobolev Institute of Mathematics

By a graph we mean a finite connected multigraph without bridges. The genus of a graph is the dimension of its homology group. Two graphs are isospectral is they share the same Laplacian spectrum. We prove that two genus two graphs are isospectral if and only if they are isomorphic. Also, we present two isospectral bridgesless genus three graphs that are not isomorphic.

Regular embeddings of complete bipartite graphs with multiple edges

Na-Er Wang

Matej Bel University

Every regular embedding $\mathcal{N}$ of a graph with multiple edges projects onto a regular embedding $\mathcal{M}$ of a simple graph (the shadow map), preserving the vertices and their adjacency relations. It turns out that the map $\mathcal{N}$ is regular covering of $\mathcal{M}$, totally branched at vertices, and the appearance of multiple edges produces much more embeddings. In this talk, as a continuation of the celebrated classification of regular embeddings of the (simple) complete bipartite graphs by G.A. Jones et al, we will consider the classification problem of regular embeddings of the complete bipartite graphs with multiple edges.

Automorphism Groups of Geometrically Represented Graphs

Peter Zeman

Charles University in Prague

We give a description of the automorphism group of an arbitrary interval graph using group products.

Enumeration of S-rings over small groups

Matan Ziv-Av

Ben-Gurion University of the Negev

A Schur ring (S-ring for short) over a group H, may be considered as a partition of the complete graph with elements of H as vertices into some Cayley graph over H with an additional coherency property. A Cayley graph is called coherent if it is a member of an S-ring.

Some of the known distance regular antipodal covers of strongly regular graphs are coherent Cayley graphs, providing a link between classification of S-rings and of antipodal covers.

Using a computer we enumerated all S-rings over groups of order no more than 60.

We will survey the current status of the effort to classify all S-rings and will discuss the results and challenges of computerized enumeration of S-rings over small groups.

Construction of large digraphs of given degree and diameter and their symmetries

Mária Ždímalová

Slovak University of Technology in Bratislava

The degree - diameter problem for graphs and digraphs deals with the determining the largest possible graphs of given degree and diameter. In this talk we discuss the directed version of the degree - diameter problem, i.e., to determine the largest order of a digraph of a given maximum out-degree and diameter. For more details see [4].

We focus on special case: vertex-transitive digraphs of a given degree and diameter. In this contribution we pesent the three construction of Faber, Moore and Chen [2], Comellas - Fiol [1] and Gómez [3] which appear to give the current largest orders of vertex - transitive digraphs for any sufficiently large degree and diameter.

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