ZÁMEČEK 2017

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Preface

This conference booklet contains abstacts and other information related to the Fourth Zámeček workshop on Analytic Combinatorics.

The workshop was held at the beautiful Hraniční Zámeček in the UNESCO heritage site of Lednice-Valtice from March 26 till April 1, 2017. The previous installments of the Zá meček workshop were in 2015, 2012 and 2009. The workshop was attended by 33 participants. It was supported jointly by grants from Amsterdam, Budapest and Prague. (AFMIDMOA , STRUCLIM project 617 747, ERC-CZ 1201 CORES and DIMATIA).

More scientific information about the developing area of structural limits in their relationship to algebra, model theory and analysis will be included in a planned special issue of the European Journal of Combinatorics.

We thank to all participants for their contribution to the workshop and particularly to my coorganisers László Lovász, Lex Schrijver, Balázs Szegedy, and to our secretary Petra Milštainová.

This booklet and cartoons were produced by Andrés Aranda López.

Jaroslav Nešetřil

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Monday

L. Lovász	Hyperfinite Graphs and Multiway Cuts
M. Abért	Amenability and Entropy
G. Regts	Zeros of the Independence Polynomial
	Walk: Tři Grácie
I. Adler	Nowhere Dense Classes and Stability
B. Sevenster	On the Skew Edge-coloring Model

For a (finite) graph $G = (V, E)$, a set $T \subseteq E$ will be called *k-splitting*, if every connected component of $G \setminus T$ has at most k nodes. We denote by $\mathcal{P}_T$ the partition of V into the connected components of $G \setminus T$. We denote by $\sigma_k(G)$ the minimum of $|T|/n$, where T is a k-splitting set. In [1], a graph G is called *(ε, k)-hyperfinite*, if $\sigma_k(G) \le \varepsilon$.

Let $\mathcal{R}$ denote the set of subsets $Y \subseteq V$ with $|Y| \le k$. We call a probability distribution τ on $\mathcal{R}$ a *fractional $\mathcal{R}$-partition*, if selecting $\mathbf{Y} \in \mathcal{R}$ according to τ, and then a point $\mathbf{y} \in \mathbf{Y}$ uniformly, we get a uniformly distributed point in V. If T is k-splitting and we select $Y \in \mathcal{P}_T$ with probability $|Y|/n$, we get such a fractional $\mathcal{R}$-partition τ_T.

For $Y \subseteq V$, let ∂Y denote the set of edges connecting Y to $V \setminus Y$. Define the *boundary value* of τ as

$$\partial(\tau) = \mathsf{E}\Big(\frac{|\partial \mathbf{Y}|}{|\mathbf{Y}|}\Big) = \sum_{Y \in \mathcal{R}} \tau(Y) \frac{|\partial Y|}{|Y|}.$$

Define

$$\sigma_k^*(G) = \frac{1}{2} \min_{\tau} \partial(\tau),$$

where τ ranges over all fractional $\mathcal{R}$-partitions. In particular, for every k-splitting set T, we have $\partial(\tau_T) = \frac{2|T|}{n}$. This implies that $\sigma_k^* \le \sigma_k$. The converse is not true, but we have the following weak converse:

Theorem 3.1 *For every finite graph G with maximum degree D,*

$$\sigma_k(G) \le \sigma_k^*(G) \log\Big(\frac{8D}{\sigma_k^*(G)}\Big).$$

The theorem generalizes to the graph limit setting. Let G be a graphing with degrees bounded by D. (In the definition of σ_k, we consider the edge-measure of Y instead of $|Y|/n$.) As a corollary, we can prove that hyperfiniteness of a graphon is invariant under weak isomorphism. This in turn implies the result of Schramm [2] that a convergent sequence of bounded-degree graphs is hyperfinite if and only if its limit graphing is hyperfinite.

The theorem can be generalized to hypergraphs. Let $\mathcal{H}$ be a hypergraph on node set V with $|V| = n$. Assume that $\{x\} \in \mathcal{H}$ for each $x \in V$. Let $w: \mathcal{H} \to \mathbb{R}_+$ be an edge-weighting such that $w(\{x\}) \le 1$ for $x \in V$. Define

$$\sigma(\mathcal{H}, w) = \frac{1}{n} \min \left\{ \sum_{Y \in \mathcal{F}} w(Y) : \ \mathcal{F} \subseteq \mathcal{H}, \ \cup \mathcal{F} = V \right\},$$

and

$$\sigma^*(\mathcal{H},w)=\frac{1}{n}\min\left\{\sum_{A\in\mathcal{H}} w(A)x_A:\ x\in\mathbb{R}_+^{\mathcal{H}},\ \sum_{A\ni x} x_A\geq 1\ (\forall x\in V)\right\}.$$

Then

$$\sigma^*(\mathcal{H},w)\leq\sigma(\mathcal{H},w)\leq\sigma^*(\mathcal{H},w)\log\frac{8}{\sigma^*(\mathcal{H},w)}.$$

References

[1] L. Lovász: *Large networks and graph limits*, Amer. Math. Soc., Providence, RI (2012).

[2] O. Schramm: Hyperfinite graph limits, *Elect. Res. Announce. Math. Sci. 15* (2008), 17–23.

Guus Regts: On a Conjecture of Sokal Concerning Roots of the Independence Polynomial

A conjecture of Sokal (2001) regarding the domain of non-vanishing for independence polynomials of graphs, states that given any natural number $\Delta\geq 3$, there exists a neighborhood in $\mathbb{C}$ of the interval $[0,\frac{(\Delta-1)^{\Delta-1}}{(\Delta-2)^{\Delta}})$ on which the independence polynomial of any graph with maximum degree at most Δ does not vanish. The main message of this talk is that this conjecture is true. In the talk I will further explain how this conjecture is related to the existence of efficient approximation algorithms for evaluating the independence polynomial. After that I will discuss some of the ideas that we used to prove the conjecture. See `https://arxiv.org/abs/1701.08049` and `https://arxiv.org/abs/1701.08049` for details.

Based on joint work with Han Peters

Isolde Adler: PAC Learning of FO Definable Concepts & Nowhere Dense Graph Classes

In machine learning, the problem of ”concept learning” is to identify an unknown set from a given concept class (i.e. collection of sets) algorithmically. In the model of ”probably approximately correct” (PAC) learning, the learner receives a number of samples and must be able to identify the unknown set approximately, in a probabilistic sense. It is well-known the the sample size for PAC learning is characterised by the Vapnik-Cervonenkis (VC) dimension of the concept class. We are interested in the VC dimension of concept classes that are definable in some logic on classes of finite graphs. In 2004, Grohe and TurÃąn showed that for any subgraph closed class C, monadic second-order definable concept classes have bounded VC dimension on C if and only if C has bounded tree-width. We show that for any subgraph closed class C, first-order definable concept classes have bounded VC dimension on C if and only if C is nowhere dense.

This is joint work with Hans Adler

Tuesday

S. Janson	On Convergence for Graphexes
R. Šámal	Vector Colorings
D. Kunszenti-Kovács	From Multigraph Limits to Banach Space Valued Graphons
J. Turner	Unistochastic Matrices and Quantum Walks
	Walk: Rybniční zámeček
B. Szgedy	Neural Networks and Graph Limits
L. Vena	On the Number of Monochromatic Configurations

Svante Janson: Convergence of Graphexes

The standard theory of graph limits is useful only for dense graphs. There have been several different variations of the theory for sparse graphs. One recent version, developed by Caron and Fox; Borgs, Chayes, Cohn and Holden; Veitch and Roy, and others, uses graphons defined on an arbitrary sigma-finite measure space (instead of a probability space as in the classical theory), which without loss of generality can be taken as the positive real line, and a minor extension of these graphons called graphexes. We study four different notions of convergence for graphexes, giving some properties of them and some relations between them. We also extend results by Veitch and Roy on convergence of empirical graphons.

Robert Šámal: Vector Coloring with Applications

We study vector (and strong vector) coloring of graphs – better known as Lovász' and Schrijver's ϑ functions. We show how to certify the uniqueness of the coloring, or more generally how to describe the set of all solutions. The main idea is so-called certification matrix, which is a solution to a dual semidefinite program. As an application we give a sufficient condition of a graph to be a core.

We also discuss the behaviour of vector coloring for (categorical) graph products: we solve the variant of Hedetniemi conjecture for these chromatic graph parameters.

Joint work with Chris Godsil, David Roberson, Brendan Rooney, and Antonios Varvitsiotis.

Lluís Vena: On the Number of Monochromatic Configurations

In this talk we present some results regarding the number of monochromatic instances found in finite Ramsey theoretical questions. We show that there are colorings of the cube $[k]^n$ with $o((k+1)^n)$ monochromatic combinatorial lines (with an exponential decay as n grows). Furthermore, we present necessary and sufficient conditions for the minimal number of monochromatic configurations to be a positive proportion of the total number of possible configurations.

Wednesday

P. Ossona de Mendez	
E. Csóka	Inequalities for Factors of IID Processes Using Cooperative Game Theory
T. Martins	Finitely Forcible Graph Limits are Universal
D. Roberson	Homomorphisms of Strongly Regular Graphs

Endre Csóka: : Inequalities for Factors of IID Processes Using Cooperative Game Theory

We show a generalization of the entropy and correlation inequalities for factor of iid processes on regular infinite trees. Our techniques use Shapley-value, which is an important concept in cooperative game theory.

Taísa Martins: Finitely Forcible Graph Limits are Universal

The theory of graph limits provide analytic tools to analyse large graphs. Large dense graphs can be represented by an analytic object called graphon. Because of its close relation to extremal combinatorics, finitely forcible graphons, i.e. graphons that are defined uniquely by finitely many subgraph densities, have been an object of intensive study. Lovasz and Szegedy conjectured that all finitely forcible graphons have simple structure in several different meanings, which is now known to be false. Here, we show that any graphon is a subgraphon of a finitely forcible graphon, which dismisses any hope for a result showing that finitely forcible graphons possess a simple structure. This is a joint work with Jacob Cooper and Dan Kral.

David E. Roberson: Homomorphisms of Strongly Regular Graphs

We prove that if G and H are primitive strongly regular graphs with the same parameters and ϕ is a homomorphism from G to H, then ϕ is either an isomorphism or a coloring (homomorphism to a complete subgraph). Therefore, the only endomorphisms of a primitive strongly regular graph are automorphisms or colorings. This confirms and strengthens a conjecture of Cameron and Kazanidis that all strongly regular graphs are cores or have complete cores. The proof of the result is based on a simple application of complementary slackness to a pair of SDPs defining the Lovasz theta number.

Thursday

O. Pikhurko	Borel Local Lemma
Á. Backhausz	On the Graph-Limits Approach to Eigenvectors of Random Regular Graphs
V. Patel	Zero-free Regions and Approximation Algorithms for Graph Polynomials
S. Polak	Upper Bounds for Nonbinary Codes Based on Divisibility Arguments
	Walk: Apolon
D. Piguet	Tilings in Graphons
A. Mészáros	Mod p Corank of Benjamini-Schramm Convergent Sequences

Oleg Pikhurko: Borel Local Lemma

The Lovasz Local Lemma is a powerful tool for finding combinatorial objects with given local constraints. We present a Borel version of the local lemma, i.e. we show that, under suitable assumptions, if the set of variables in the local lemma has a structure of a Borel space, then there exists a satisfying assignment which is a Borel function. The main tool which we develop for the proof, which is of independent interest, is a parallel version of the Moser-Tardos algorithm which uses the same random bits to resample clauses that are far enough in the dependency graph.

This is joint work with Endre Csóka, Łukasz Grabowski, András Máthé and Konstantinos Tyros

Ágnes Backhausz: On the Graph Limit Approach to Eigenvectors of Random Regular Graphs

The talk is about the eigenvectors of a randomly chosen d-regular graph on n vertices. Analogous problems for dense random graphs have been studied in the last years, using the techniques of random matrix theory. To deal with the case of sparse matrices (bounded degree graphs), we used graph limit theory. Our result is about the empirical distribution of an arbitrary eigenvector of the adjacency matrix of the graph. We could prove that this empirical distribution is close to an appropriate Gaussian distribution if the number of vertices is large enough. In the talk we also discuss how this is related to the delocalization problem of eigenvectors, and what we get for approximate eigenvectors. Finally, the main ideas of the proof are presented. Joint work with Balazs Szegedy.

Sven Polak: New nonbinary code bounds based on divisibility arguments

Fix $n, q \in \mathbb{N}$. A *word* is an element $v \in [q]^n := \{1, 2, \ldots, q\}^n$. For two words $u, v \in [q]^n$, we define their *(Hamming) distance* $d_H(u, v)$ to be the number of i with $u_i \neq v_i$. A *code* is a subset of $[q]^n$. For any code $C \subseteq [q]^n$, the minimum distance $d_{\min}(C)$ of C is the minimum distance between any pair of distinct code words in C. If $C \subseteq [q]^n$ has minimum distance at least d, we call C an $(n, d)_q$ code Now we define, for a natural number d,

$$A_q(n,d) := \max\{|C| \mid C \text{ is an } (n,d)_q\text{-code }\}. \tag{1}$$

The parameter $A_q(n,d)$ is often hard to compute. It is the stable set number of the graph $G = (V,E)$ with $V := [q]^n$ and $E := \{\{u,v\} \ : \ 0 < d_H(u,v) < d\}$.

In this talk we will consider the case $(n,d)_q = (8,6)_5$. It is known that $A_5(7,6) = 15$ and that $(7,6)_5$-codes of size 15 are in 1-1-relation with solutions to Kirkman's school girl problem. These results imply that $A_5(8,6) \leq 75$. (To see this, note that in a $(8,6)_5$ code C of size more than 75 a symbol would be contained > 15 times in the first column. Consider the > 15 words in C that have this symbol in the first column. The remaining 7 columns of these words would form a $(7,6)_5$-code of size > 15.)

We will show that an $(8,6)_5$ code of size 75 cannot exist, using a divisibility argument based on occurences of symbols in each column in a code of maximum size. Further exploiting the divisibility argument yields $A_5(8,6) \leq 65$. Finally, we explain that the argument can be used to improve upper bounds on $A_q(n,d)$ for other q,n,d.

References

[1] S. C. Polak, New nonbinary code bounds based on divisibility arguments, *to be published in Designs, Codes and Cryptography*, 2017.

Diana Piguet: Tilings in Graphons

We present an analogue of graph tilings in the context of graphons. A naive analogue of matching for a graphon $W : \Omega^2 \to [0,1]$ would be to allow at most one element $y \in \Omega$ for each $x \in \Omega$ with the property that $W(x,y) > 0$. However, as any concept for graphons should not depend on a zero-measure set, this naive straightforward analogue does not make sense. Instead we take a fractional approach, which we present in a somewhat more general notion of H-tilings, i.e., finding vertex-disjoint copies of a fixed graph H in the host graph (taking $H = K_2$, we recover the original concept). We can afford to relax the concept to that of fractional tilings, as a fractional tiling in a cluster graph can be turned into an integral tiling in the original graph using Blow-up lemma type methods.

For a fixed graph H on m vertices, we write $W^{\otimes H}$ for a function defined by

$$W^{\otimes H}(x_1, ..., x_m) = \prod_{1 \le i < j \le m, ij \in E(F)} W(x_i, x_j).$$

Then an H-tiling in a graphon $W : \Omega^2 \to [0,1]$ is a mapping $t : \Omega^m \to [0, +\infty)$ that satisfies:

1. $\mathrm{supp}(t) \subseteq \mathrm{supp}(W \otimes H)$
2. for each $x \in \Omega$ we have

$$\sum_{\ell=1}^{m} \int t(x1, \ldots, x_{\ell-1}, x, x_{\ell+1}, \ldots, x_m) \le 1.$$

The size of an H-tiling is the value of its integral and the tiling number of a graphon W is the supremum of the sizes of tilings, going through all possible H-tilings in W. We observe that the notion of tiling number is not continuous, but lower semi-continuous, which is the most useful half of continuity (at least from an extremal graph theory point of view).

Similarly as in finite graph theory, there is a dual notion of fractional tiling. A fractional H-cover of a graphon $W : \Omega^2 \to [0,1]$ is a mapping $c : \Omega \to [0,1]$ such that the measure of the set

$$(\mathrm{supp} W^{\otimes H}) \cap \{(x1, ..., xm) \in \Omega^m : \sum_{\ell-1}^{m} c(x_i) < 1\}$$

is zero. The size of a fractional cover is its integral and the fractional cover number of W is the infimum of the sizes over all possible fractional covers on W.

As for the tiling, the fractional cover-number is (only) lower semi-continuous and the duality says that the tiling number is equal to the fractional cover number.

Using these new concepts and propositions, we prove a graphon analogue of Komlós tiling theorem, which implies the original theorem in the world of finite graphs. In addition we show stability of this theorem.

The corresponding papers are available at arXiv: papers 1606.03113 and 1607.08415. Joint work with Jan Hladký and Ping Hu.

András Mészáros: The mod p co-rank of random d-regular graphs

For a fixed d we consider the following random d-regular bipartite graph G_n on $2n$ vertices. The vertex set of G_n is the disjoint union of two n element sets S and T. The random graph is defined as the union of d independent uniform random perfect matching between S and T.

Consider $d \geq 3$ and a prime p. Let A_n be the adjacency matrix of G_n considered as a matrix over the p element finite field. The random variable $\dim \ker A_n$ has a limit distribution in the following sense.

If p does not divide d, then for every $k \geq 0$ we have

$$\lim_{n\to\infty} P(\dim\ker A_n = 2k) = \mu_p(k).$$

If p divides d, then $P(\dim\ker A_n = 0)$ is 0 for every n, and for every $k \geq 1$ we have

$$\lim_{n\to\infty} P(\dim\ker A_n = 2k) = \mu_p(k-1),$$

where

$$\mu_p(k) = \frac{1}{p^{k^2}\left(1-\frac{1}{p^1}\right)^2\left(1-\frac{1}{p^2}\right)^2\cdots\left(1-\frac{1}{p^k}\right)^2}\prod_{r=1}^{\infty}\left(1-\frac{1}{p^r}\right).$$

(Note that $\dim\ker A_n$ is always even.)

If $p > 2$ a similar statement is true for the non-bipartite random d-regular graph model, where we take d independent uniform random perfect matching.

In both cases the limit distributions have already appeared in earlier results as the limit distributions of the co-rank of certain dense random matrices.

Friday

G. Kun	
J. Hubička	Ramsey Properties of Hrushovski Constructions
B. Litjens	Counting Flows in Embedded Graphs
T. Hubai	Graph 3-profiles

Bart Litjens: Counting flows in embedded graphs

Let $n \in \mathbb{N}$ and let $\mathbb{Z}_n$ denote the cyclic group of order n. The notion of a nowhere-zero $\mathbb{Z}_n$-flow on a graph $\Gamma = (V, E)$ can be generalized to nowhere-identity G-flows, where G is any finite group, when a rotation system for Γ is given (for each $v \in V$ a cyclic order of the edges incident with v). This leads to consider flows in graphs that are embedded in a closed surface (i.e., maps). The number of nowhere-identity G-flows depends on the topology of the surface and the multiset of dimensions of irreducible representations of G. It is a specialization of a (new) polynomial invariant of maps, called the surface Tutte polynomial. Joint work with Andrew Goodall, Guus Regts and Lluís Vena

Pictures

genvalues and let $b_1, \ldots, b_n$
eigenvalue distribution
$\mu_G = \frac{1}{n}\sum_{i=1}^{n} \delta_{\lambda_i}$
rac measure concentrated on
No effect!
Maybe with this Liquid...
Any questions?


Kolonáda
Rajbna 2/9/99