ITI Series (IUUK)

Institut teoretické informatiky
Institute for Theoretical
Computer Science

Informatický ústav Univerzity Karlovy
Computer Science Institute
of Charles University

I T I

I U U K

2022-685

Petr Gregor, Jan Kratochvíl, Martin Loebl, Martin Pergel
(eds.)

CSGT 2022

Institute for Theoretical
Computer Science (ITI)
Charles University

Computer Science Institute
of Charles University
(IUUK)

Malostranské náměstí 25
118 00 Praha 1
Czech Republic

http://iti.mff.cuni.cz/series/

http://iuuk.mff.cuni.cz/series/

8th Czech-Slovak International Symposium on Graph Theory, Combinatorics, Algorithms and Applications

Dedicated to the Memory of Robin Thomas

July 25-29, 2022, Prague, Czech Republic

Booklet of Abstracts

Petr Gregor, Jan Kratochvíl, Martin Loebl, Martin Pergel (eds.)

The 8th Czech-Slovak International Symposium on Graph Theory, Combinatorics, Algorithms and Applications is organized by

in cooperation with

Main partner

Partner

Sponsor

Local arrangements in cooperation with CONFORG, s.r.o.

Published by ITI Series (IUUK) 2022-685 & MATFYZPRESS as the 662th publication
Computer Science Institute of Charles University, IUUK
Faculty of Mathematics and Physics, Charles University
Malostranské nám. 25, 118 00 Prague 1, Czech Republic

Prague 2022

ISBN 978-80-7378-468-3

Preface

Dear participants,

welcome to the 8th Czech-Slovak International Symposium on Combinatorics, Graph Theory, Algorithms and Applications which is organized by the Department of Applied Mathematics of the Charles University in Prague, in cooperation with the universities from Bratislava, Brno, Košice, Ostrava, and Plzeň. We are happy to see so many registered participants from all around the world. And we are happy that after the two years affected by COVID-19 we can host an (almost) fully physical on-site event.

With its 60 years long tradition, the Czech-Slovak Graph Theory belongs to the longest running mathematical conference series, having been organized every year since its first issue in Smolenice in 1963. Roughly every 8 years the event is organized as a large international symposium. And one of these is this year conference which offers you 11 plenary and 70 contributed talks.

Our conference is dedicated to the memory of Robin Thomas, one of the most celebrated Czech mathematicians. Robin graduated from Charles University in 1985, received the Fulkerson Prize twice, was named the SIAM fellow and a fellow of AMS. Being affiliated with Georgia Institute of Technology in Atlanta, he has kept strong links with the Czech and Slovak graph theory communities, e.g., delivered plenary lectures at the Conference of Czech Mathematicians in 2002 and at the 6th CSGT in 2006. Robin passed away in the spring of 2020. The choice of plenary speakers includes Robin's students, frequent collaborators, and his former advisor. Jaroslav Nešetřil and Paul Wollan will deliver special Borůvka and Jarník lectures organized in the historial building of Charles University on Wednesday afternoon.

A special issue of Discrete Mathematics stemming from the symposium will be guest edited by Jan Kratochvíl, Martin Loebl and Jaroslav Nešetřil. Selected contributions based on original so far unpublished results will be solicited during the symposium.

Last but not least, we would like to thank the partners of the symposium, RSJ financial group and Avast, for substantial financial help, and to our sponsor Znovín Znojmo for a liquid contribution to the conference dinner.

We hope that you will have a good time in Prague, and fully enjoy the scientific and social program of the symposium, as well as the atmosphere of the city of Prague.

Jan Kratochvíl and Martin Loebl
Organizing committee and Program committee chairs

Life and Work of Robin Thomas

Robin Thomas studied at the Faculty of Mathematics and Physics, Charles University, where he completed a master's degree in Mathematical Analysis in 1985 and earned his doctorate in 1992. His course of study was greatly influenced by his advisor, Prof. Jaroslav Nešetřil.

Shortly before the Velvet Revolution in 1989, he took a position at Ohio State University in Columbus, and after two years moved to Georgia Institute of Technology in Atlanta. Georgia Tech became his home institution for the rest of his life.

Robin Thomas did not confine his scientific work to a narrow field, studying as he did questions in combinatorics, algebra, geometry, topology, and theoretical computer science. But the core of his research lay in structural graph theory. It was in this area that he published most extensively, solving many of its famous problems and influencing its development as a discipline. He was, for example, instrumental in verifying the solution of the Four Color Theorem for planar graphs in the 1990s. As a teacher, he raised many excellent mathematicians who now work at prestigious universities worldwide.

Robin Thomas received major international honors for his research. He was twice awarded the Fulkerson Prize for outstanding papers in the area of discrete mathematics, in 1994 and in 2009. In 2012 he was named Fellow of the American Mathematical Society (AMS), and in 2018 he was named Fellow of the Society for Industrial and Applied Mathematics (SIAM). He gave many invited lectures at international conferences, including the International Congress of Mathematicians in Madrid, 2006. His work was recognized in his home country, too. As a coauthor, he received the Prize of the Rector of Charles University for the Best Publication in the Natural Sciences in 2006, and in 2011 he was awarded the Neuron Prize for Contribution to World Science in Mathematics.

Robin Thomas never lost contact nor ceased cooperation with his former teachers and classmates (now professors and teachers themselves) back in the Czech Republic. Several of our colleagues spent a study visit or a postdoc under his supervision. These include Daniel Kráľ, now professor at the Faculty of Informatics of Masaryk University in Brno, and Zdeněk Dvořák, professor at the Computer Science Institute of Charles University, both of whom continued to work closely with him after the end of their stays at Georgia Tech and were among his most frequent coauthors.

Robin Thomas was an outstanding scientific personality and an outstanding human being.

Jan Kratochvíl, Martin Loebl, Jaroslav Nešetřil

Table of Contents

Part I

Plenary Talks

Induced subgraphs and small tree decompositions

Maria Chudnovsky

Princeton University, Princeton, NJ, USA

Tree decompositions are a powerful tool in structural graph theory; they are traditionally used in the context of forbidden graph minors. Connecting tree decompositions and forbidden induced subgraphs has until recently remained out of reach.

Tree decompositions are closely related to the existence of "laminar collections of separations"in a graph, which roughly means that the separations in the collection "cooperate" with each other, and the pieces that are obtained when the graph is simultaneously decomposed by all the separations in the collection "line up" to form a tree structure. Such collections of separations come up naturally in the context of forbidden minors.

In the case of families where induced subgraphs are excluded, while there are often natural separations, they are usually very far from forming a laminar collection. However, under certain circumstances, these collections of natural separations can be partitioned into a small number of laminar collections (in this context "small"means either constant or logarithmic in the number of vertices of the graph). This in turn allows us to obtain a wide variety of structural and algorithmic results, which we will discuss in this talk.

Flows and coloring of near-quadrangulations

Zdeněk Dvořák

Charles University, Prague

The near-quadrangulations of surfaces (graphs where almost all faces have length four) play an important role in the theory of 3-colorability of triangle-free graphs on surfaces. We present a powerful approach to coloring near-quadrangulations using nowhere-zero flows and explore its applications.

Recent progress on Directed Graph Minor

Ken-ichi Kawarabayashi

National Institute of Informatics and U. Tokyo

Graph Minor project by Robertson and Seymour is perhaps the deepest theory in Graph Theory. It gives a deep structural characterization of graphs without any graph as a minor. It also gives many exciting algorithmic consequences.

10 years ago, with Stephan Kreutzer (and his students/PDs), we start extending Graph Minor Theory to directed graphs. There is some progress, but many things are to be done. In this talk, we present some progress report. Topics include

1. The directed grid theorem
2. The directed flat wall theorem
3. Tangle tree decomposition
4. Variant of the directed disjoint paths problems
5. Testing flatness
6. Toward the structure (and decomposition) theorem for H-minor-free digraphs.

Common structures in combinatorics

Daniel Kráľ

Faculty of Informatics, Masaryk University, Brno

Ramsey's Theorem guarantees the existence of monochromatic substructures in sufficiently large colored combinatorial structures, e.g., graphs. We consider the following quantitative version of this problem: a combinatorial structure is *common* if the random coloring minimizes the number of its monochromatic copies. The notion of common graphs can be traced back to the work of Erdős from the 1960s. In particular, Erdős conjectured that every complete graph is common, which was disproved by Thomason in the 1980s.

A classification of common graphs remains a challenging open problem. Sidorenko's Conjecture, one of the most significant open problems in extremal graph theory, implies that every 2-chromatic graph is common. While examples of 3-chromatic common graphs were known for a long time, the existence of a 4-chromatic common graph was open until 2012, and no common graph with a larger chromatic number is known.

In this talk, we will survey some recent results on common combinatorial structures, specifically focusing on common graphs. In particular, we will discuss a construction of connected common graphs with arbitrarily large chromatic number, the notion of locally common graphs studied by Csóka, Hubai and Lovász, the notion of common structures in algebraic settings, and extensions to colorings with more than two colors.

The talk will include results obtained with different groups of collaborators, including Robert Hancock, Matjaž Krnc, Ander Lamaison, Jonathan A. Noel, Sergey Norin, Péter Pál Pach, Jan Volec and Fan Wei.

Building the hierarchy of graph classes

Sang-il Oum

IBS Discrete Mathematics Group, Daejeon and KAIST, Daejeon

We will give a survey on the classification of graph classes in terms of the transductions in monadic second-order logic. Blumensath and Courcelle [1] characterized that every class of graphs is equivalent by transductions of the monadic second-order logic of the second kind to one of the following: class of all trees of height n for an integer n, class of all trees, class of all paths, and class of all grids. They conjectured that there is a similar linear hierarchy of graph classes in terms of the monadic second-order logic of the first kind. We will discuss how a recent theorem of the speaker with O-joung Kwon, Rose McCarty, and Paul Wollan [3] on the vertex-minor obstruction for shrub-depth and a theorem of the speaker with Bruno Courcelle [2] on graphs of large rank-width and logical expression of vertex-minors solve some subproblems of their conjecture.

Reference

[1] A. Blumensath and B. Courcelle. On the monadic second-order transduction hierarchy. *Log. Methods Comput. Sci.*, 6(2):2:2, 28, 2010.

[2] B. Courcelle and S. Oum. Vertex-minors, monadic second-order logic, and a conjecture by Seese. *J. Combin. Theory Ser. B*, 97(1):91–126, 2007.

[3] O. Kwon, R. McCarty, S. Oum, and P. Wollan. Obstructions for bounded shrub-depth and rank-depth. *J. Combin. Theory Ser. B*, 149:76–91, 2021.

Perfect matchings in cubic graphs

Edita Máčajová

Comenius University, Bratislava

It is known that many important conjectures in graph theory are equivalent to their restrictions to cubic graphs. For better understanding of this family, perfect matchings turn out to be of vital importance.

In this talk, we provide general background concerning the most important conjectures that involve perfect matchings in cubic graphs and their interactions. These conjectures include (1) conjectures about covering the edge set of a cubic graph with a certain constant number of perfect matchings (Berge conjecture), (2) conjectures about covering the edges of a cubic graph with perfect matchings in such a way that every edge is in the same number of perfect matchings (Fulkerson conjecture), (3) conjectures about the existence of a certain constant such that every graph admits this number of perfect matchings with empty intersection (Fan-Raspaud conjecture).

We discuss a special family of subcubic graphs not containing a perfect matching and its consequences for cubic graphs. We also sketch the proof of the statement that every bridgeless cubic graph contains two perfect matchings whose complement is bipartite. This statement is implied by the Fan-Raspaud conjecture and was conjectured to be true 10 years ago by Giuseppe Mazzuoccolo.

This talk is based on joint works with Karabáš, Kardoš, Nedela, Škoviera, Zerafa.

Tree models for graphs

Jaroslav Nešetřil

Computer Science Institute, Faculty of Mathematics and Physics, Charles University

We survey various tree models starting with Tremaux and Boruvka and ending with shrub depth and tree models for permutation and twin width classes.

Approximation of Submodular Minimum Linear Ordering Problems

Prasad Tetali

Carnegie Mellon University, Pittsburgh

The minimum linear ordering problem (MLOP) generalizes well-known combinatorial optimization problems such as minimum linear arrangement and the minimum sum set cover. MLOP seeks to minimize an aggregated cost $f(\cdot)$ due to an ordering σ of the items (say $[n]$), i.e., $\min_\sigma \sum_{i\in[n]} f(E_{i,\sigma})$, where $E_{i,\sigma}$ is the set of items mapped by σ to indices $[i]$. Besides reviewing past results on the problem and its special cases due to Feige, Iwata, Lovasz, and the speaker, we will outline new results. These include a new combinatorial algorithm for approximating monotone submodular MLOP, using the theory of principal partitions, resulting in a $2 - \frac{1+\ell_f}{1+|E|}$ approximation for monotone submodular MLOP, where $\ell_f = \frac{f(E)}{\max_{x\in E} f(\{x\})}$ satisfies $1 \leq \ell_f \leq |E|$. Our theory provides new approximation bounds for special cases of the problem, in particular a $2 - \frac{1+r(E)}{1+|E|}$ approximation for the matroid MLOP, where $f = r$ is the rank function of a matroid. We further observe that the minimum latency vertex cover is $\frac{4}{3}$ approximable, by which we also lower bound the integrality gap of its natural LP relaxation. These are obtained in joint work with Majid Farhadi, Swati Gupta, Shengding Sun and Michael Wigal, all colleagues from Georgia Tech.

From even-hole free graphs to treewidth

Nicolas Trotignon

CNRS, ENS de Lyon, Lyon

A *hole* in a graph is a chordless cycle of length at least 4. A graph is *even hole free* if it does not contain a hole of even length. Even-hole-free graphs attracted some attention because of their anology with perfect graphs (where holes of odd length together with their complements are excluded). We will survey some results about the structure of even-hole-free graphs, and explain why their structure is so mysterious : on the one hand, the general theorems that are known about them are not strong enough to provide polynomial time algorithms to color or find a maximum stable set; one the other hand, it is quite hard to exhibit example of even hole free that are "complex" in any way.

This remark lead researchers to investigate widths of even-hole-free graphs (treewidth, cliquewidth, rankwidth), with the hope that in some way their width might be restricted. This study failed in the sense that very restricted classes of even-hole graphs turned out to have unbounded width. But it was a success in the sense it lead to several conjectures about a possible version of the celebrated grid-minor theorem of Roberston and Seymour, with "minor" replaced by "induced subgraph". It turns out that all these conjectures are now either proved or disproved. We will survey these recent progress.

Linear programming and the circuit imbalance measure

László Végh

London School of Economics

The existence of a strongly polynomial algorithm for linear programming (LP) is a fundamental open question in optimization. Given an LP in the standard equality form

$$\langle c, x\rangle \text{ s.t. } Ax = b\,,\ x \geq 0\,,$$

for $A \in \mathbb{R}^{n\times n}$, $b \in \mathbb{R}^m$, $c \in \mathbb{R}^n$, such an algorithm would perform $\text{poly}(n, m)$ arithmetic operations. Strongly polynomial algorithms are known for a range of network optimization problems. Two significant steps towards general LP are Tardos's $\text{poly}(n, m, \log \Delta_A)$ algorithm from 1986 and a $\text{poly}(n, m, \log \bar{\chi}_A)$ interior point method by Vavasis and Ye from 1996. Here, Δ_A is the maximum subdeterminant of the integer constraint matrix, and $\bar{\chi}_A$ is a geometric condition number associated with the matrix A.

We give an overview of recent developments that strengthen and extend these results. A key object of our study is the *circuit imbalance measure* κ_A that bounds the ratios of the entries of support-minimal vectors in the kernel of A. We exhibit combinatorial properties and proximity results of linear programs that can be used to design new exact LP algorithms. In particular, we present new circuit augmentation algorithms, and derive improved bounds on the circuit diameter of polyhedra.

The talk is based on joint works with Daniel Dadush, Sophie Huiberts, Cedric Koh, and Bento Natura.

Explicit bounds for graph minors

Ken-ichi Kawarabayashi, Robin Thomas, Paul Wollan

Sapienza University of Rome, Rome

Robertson and Seymopur proved a theorem approximately characterizing all graphs excluding some fixed graph H as a minor, a result which has had an enormous impact on the field with numerous applications in graph theory and theoretical computer science. The proof is notable for its complexity, stretching over a series of sixteen papers. Moreover, the proof does not yield explicit bounds on the parameters involved.

We present recent work which for the first time gives explicit bounds on the parameters involved through a new proof of the main results in the graph minors series.

Part II

Contributed Talks

From domination to isolation of graphs

Peter Borg

University of Malta, Malta

In 2017, Caro and Hansberg [6] introduced the isolation problem, which generalizes the domination problem. Given a graph G and a set $\mathcal{F}$ of graphs, the *$\mathcal{F}$-isolation number of* G is the size of a smallest subset D of the vertex set of G such that $G - N[D]$ (the graph obtained from G by removing the closed neighbourhood of D) does not contain a copy of a graph in $\mathcal{F}$. When $\mathcal{F}$ consists of a 1-clique, the $\mathcal{F}$-isolation number is the domination number. Caro and Hansberg [6] obtained many results on the $\mathcal{F}$-isolation number, and they asked for the best possible upper bound on the $\mathcal{F}$-isolation number for the case where $\mathcal{F}$ consists of a k-clique and for the case where $\mathcal{F}$ is the set of cycles. The solutions [1, 3] to these problems will be presented together with other results, including an extension of Chvátal's Art Gallery Theorem. Some of this work was done jointly with Kurt Fenech and Pawaton Kaemawichanurat.

Reference

[1] P. Borg, Isolation of cycles, Graphs and Combinatorics 36 (2020), 631–637.

[2] P. Borg, Isolation of connected graphs, arXiv:2110.03773.

[3] P. Borg, K. Fenech and P. Kaemawichanurat, Isolation of k-cliques, Discrete Mathematics 343 (2020), paper 111879.

[4] P. Borg, K. Fenech and P. Kaemawichanurat, Isolation of k-cliques II, Discrete Mathematics 345 (2022), paper 112641.

[5] P. Borg and P. Kaemawichanurat, Domination and partial domination of maximal outerplanar graphs, arXiv:2002.06014.

[6] Y. Caro and A. Hansberg, Partial domination - the isolation number of a graph, FiloMath 31:12 (2017), 3925–3944.

Geometry meets Topology at Optimal RAC Graphs

Franz J. Brandenburg

Passau, Germany

A drawing of a graph in the plane is *right angle crossing* (*RAC*) if the edges are drawn straight line and may cross at a right angle. A drawing or embedding is *1-planar* if each edge is crossed at most once. A graph is 1-planar (RAC) if it admits a 1-planar (RAC) drawing.

It is known that an n-vertex graph has at most 4n-8 edges if it is 1-planar and at most 4n-10 edges if it is RAC. Both bounds are tight. An n-vertex 1-planar (RAC) graph is *optimal* if it has 4n-8 (4n-10) edges.

It has been shown that there are optimal 1-planar graphs for $n = 8$ and for every $n \geq 10$, that optimal 1-planar graphs can be recognized in linear time, and that every optimal RAC graph is 1-planar.

Our contribution: We call a graph *almost optimal RAC* if it admits a 1-planar drawing such that the restrictions to straight-line edges and right angle crossings are relaxed. We show that there are (almost) optimal RAC graphs for every $n \geq 4$. Every (almost) optimal RAC graph has a unique 1-planar embedding except for so called doubly linked paths, K_4 and K_5. Almost optimal RAC graphs can be recognized in cubic time, whereas the recognition problem for optimal RAC graphs remains open.

Acknowledgement. Supported by Deutsche Forschungsgemeinschaft (DFG) Br835/20-1.

Generalizing nowhere dense graph classes

Samuel Braunfeld, Michael Laskowski

Charles University

Nowhere dense graph classes, introduced by Nešetřil and Ossona de Mendez, have proven to be a key dividing line separating tame from wild behavior in monotone (i.e. closed under subgraph) graph classes. We will discuss ongoing work to generalize this dividing line to hereditary classes of relational structures, guided by model theory.

Reference

[1] Samuel Braunfeld and Michael C. Laskowski, Characterizations of Monadic NIP, Transactions of the AMS, Series B 8 (2021), 948–970.

Counting well-quasi-ordered down-sets

Rutger Campbell, Dillon Mayhew

Institute for Basic Science, Daejeon, South Korea

For a poset consisting of combinatorial objects under some substructure relation, we characterize when there are countably many well-quasi-ordered down-sets.

On the Hyperopic Version of the Cops and Robber Game

Nancy E. Clarke, Stephen Finbow, Margaret-Ellen Messinger, Amanda Porter

Acadia University, Wolfville, Canada

We consider the hyperopic version of the Cops and Robber game introduced by Bonato et al. [1], a variation in which the robber is invisible to the cop side unless outside the neighbourhood of a cop. The hyperopic copnumber is analogous to the copnumber. We present a variety of results on this parameter for various classes of graphs, including Cartesian products and graphs of diameter 2.

Reference

[1] A. Bonato, N.E. Clarke, D. Cox, S. Finbow, F. Mc Inerney, M.E. Messinger, Hyperopic cops and robbers, Theoretical Comp. Sci. 794 (2018) 59–68.

Graphs with every hole the same length

Linda Cook, Jake Horsfield, Myriam Preissmann, Cléophée Robin, Paul Seymour, Ni Luh Dewi Sintiari, Nicolas Trotignon, and Kristina Vušković

Institute for Basic Science, Daejeon, Republic of Korea

We call an induced cycle of length at least four a hole. The parity of a hole is the parity of its length. Forbidding holes of certain types in a graph has deep structural implications. In 2006, Chudnovksy, Seymour, Robertson, and Thomas famously proved that a graph is perfect if and only if it does not contain an odd hole or a complement of an odd hole. In 2002, Conforti, Cornuéjols, Kapoor, and Vuškovíc provided a structural description of the class of even-hole-free graphs. I will describe the structure of all graphs that contain only holes of length ℓ for every $\ell \geq 7$.

This is joint work with Jake Horsfield, Myriam Preissmann, Paul Seymour, Ni Luh Dewi Sintiari, Cléophée Robin, Nicolas Trotignon, and Kristina Vušković [2]. Note that Jake Horsfield, Myriam Preissmann, Ni Luh Dewi Sintiari, Cléophée Robin, Nicolas Trotignon, and Kristina Vušković also wrote up their version of the proof in [1].

Reference

[1] Jake Horsfield, Myriam Preissmann, Cléophée Robin, Ni Luh Dewi Sintiari, Nicolas Trotignon, and Kristina Vušković. "When all holes have the same length."arXiv preprint arXiv:2203.11571 (2022).

[2] Linda Cook, Jake Horsfield, Myriam Preissmann, Cléophée Robin, Paul Seymour, Ni Luh Dewi Sintiari, Nicolas Trotignon, and Kristina Vušković. "Graphs with all holes the same length."arXiv preprint arXiv:2110.09970 (2021).

Some Applications of Voltage Graphs in Computer Science

Peter Czimmermann

University of Žilina, Žilina

A uniformly deployed set (UDS) with parameters (n, p, t) is the set that contains n-bit binary words with weights p. Each pair of such words has at most t overlapping ones. It has been shown that UDS can be used in various heuristic algorithms for searching a weighted p-median or p-center. In our previous work, we suggested a fast algorithm for construction of UDS with given parameters. This algorithm uses abelian lifts from voltage graphs for the construction of large digraphs with given properties. Rows of the adjacency matrix of such digraphs are the elements of UDS.

In our contribution, we show the construction of the family of UDS for $p = 1, 2, \ldots, n-1$ (where n is given). We study usage of this collection in heuristic algorithms that are used to find suboptimal solutions of some NP-hard problems (for example covering problem). We also introduce applications of collections of UDS in some areas of computer science – in reliability theory and artificial neural networks.

On necessary and sufficient condition for the existence of properly colored regular factors

Roman Čada, Michitaka Furuya, Kenji Kimura, Kenta Ozeki,
Christopher Purcell, Takamasa Yashima

University of West Bohemia, Plzeň

Let $r \geq 2$ be an integer, and G be a graph. A spanning subgraph of G is called a factor F of G, and if F is r-regular, then we call it r-factor. Tutte [1] gave a necessary and sufficient condition for the existence of an r-regular factor in G.

Let now G be an edge-colored graph. A subgraph is called properly colored if for each $x \in V(G)$, no pair of colors on the edges incident with x are the same. As a generalization of the Tutte's result we give a necessary and sufficient condition for the existence of a properly colored r-factor in edge-colored graphs. We will also discuss some related results.

Reference

[1] W. T. Tutte, The factors of graphs, Can. J. Math., 4 (1952), 314–328.

Light graphs in essentially-highly-connected polyhedral graphs

Katarína Čekanová, Tomáš Madaras

P. J. Šafárik University, Košice, Slovakia

A k-connected graph is called essentially $(k+1)$-connected if each its vertex k-cut leaves at most one nontrivial component. We say that graph H is light in a graph family $\mathcal{G}$ if there exist finite number k such that each $G \in \mathcal{G}$ which contains H as a subgraph, also contains its isomorphic copy K with $\sum\limits_{x \in V(K)} \deg_G(x) \leq k$.

We explore the structure of light graphs in essentially 4- and 5-connected plane graphs, focusing on existence of small clusters of faces of small sizes as well as the small subgraphs (or sets of subgraphs) having vertices of degrees upper bounded by small constants; as an application, we show that the cyclic edge connectivity of essentially 5-connected plane graphs is finite.

Complexity of Ordered Homomorphisms

Michal Čertík, Jaroslav Nešetřil

Computer Science Institute, Faculty of Mathematics and Physics, Charles University, Prague, Czech Republic

Ordered Homomorphism $f^<$ of *Ordered Graphs*, graphs with linearly ordered set of vertices, $G^< = (V, E, <^G)$ and $H^< = (V, E, <^G)$ is a homomorphism $f : G \to H$, for which $v <^G u \implies f(v) <^H f(u)$. This homomorphism can be seen as a decomposition of vertices of $G^<$ into k intervals in the order $<^G$, where each interval is independent set.

Let $\chi^<(G^<) = \min(i|$ there exists $f^< : G^< \to K_i^<)$, where $K_i^<$ is (ordered) complete graph on i vertices. It is easy to see that determining the $\chi^<(G^<)$ is in $\mathcal{P}$. This is contrary to the similar concept of standard graphs coloring, where for $i \geq 3$ the problem is $\mathcal{NP}$-complete. $\chi^<(G^<)$ can be moreover obtained by a greedy algorithm.

As for graphs [2], one can consider $H^<$-coloring problem as a question whether for a given $G^<$ there exists $f^< : G^< \to H^<$. Note that for every $H^<$, the problem is in $\mathcal{P}$.

Thus, we consider more general problem which we call $\mathcal{H}^<$*-coloring* as the following problem.

- **Instance**: Ordered graph $G^<$.
- **Question**: Does there exist $H^< \in \mathcal{H}^<$ and homomorphism $f^< : G^< \to H^<$?

We prove the following two results related to the (parameterized) complexity of this problem (see [3]).

Theorem 1. *There exists $\mathcal{H}^<$ such that $\mathcal{H}^<$-coloring is $\mathcal{NP} -$ complete.*

Theorem 2. *There exists parameter k such that $\mathcal{H}_k^<$-coloring is in $\mathcal{XP}$ and $\mathcal{W}$[2]-hard.*

The proofs are contained in the forthcoming article [1].

Reference

[1] M. Čertík and J. Nešetřil, Complexity of Ordered Homomorphisms, manuscript.

[2] P. Hell and J. Nešetřil, On the Complexity of H-Coloring, Journal of Combinatorial Theory (1986), 92–110.

[3] R. Niedermeier, Invitation to Fixed-Parameter Algorithms, Oxford University Press (2006).

Group supermagic labelings of Cartesian product of cycles

Dalibor Froncek, Peter Paananen, Lincoln Sorensen

University of Minnesota Duluth

There is a close connection between Abelian groups and Cartesian products of cycles, since every Cartesian product $C_{n_1} \Box C_{n_2} \Box \cdots \Box C_{n_t}$ can be viewed as the Cayley graph of the group $Z_{n_1} \oplus Z_{n_2} \oplus \cdots \oplus Z_{n_t}$ with generating set $\{(1,0,\ldots,0),(0,1,0,\ldots,0),\ldots,(0,0,\ldots,1)\}$.

A graph $G = (V, E)$ is Γ-*supermagic* if there exists a bijection f from E to a group Γ of order $|E|$ (called a Γ-*supermagic labeling*) such that the *weight* $w(x)$ of each vertex x, defined as the sum of labels of all edges incident with x, is equal to the same *magic element* μ. That is, there exists $\mu \in \Gamma$ such that for all $x \in V$,

$$w(x) = \sum_{xy \in E} f(xy) = \mu.$$

It was proved by DF, McKeown, McKeown, and McKeown ([1], [2]) that a Z_{2mn}-supermagic labeling of $C_m \Box C_n$ exists for all $m, n \geq 3$. We prove that when $m \equiv n \pmod 2$, then $C_m \Box C_n$ allows a Γ-supermagic labeling by any Abelian group Γ of order $2mn$. We also present some preliminary results on labelings of $C_m \Box C_n$ by non-Abelian groups, namely dihedral groups D_{mn}.

Reference

[1] D. Froncek, J. McKeown, J. McKeown, M. McKeown. Z_{2nm}-supermagic labeling of $C_n \Box C_m$, Indones. J. Combin. 2 (2018), 57–71.

[2] D. Froncek, M. McKeown, Note on diagonal construction of Z_{2nm}-supermagic labeling of $C_n \Box C_m$, AKCE Int. J. Graphs Comb. 17 (2020), 952–954.

Stable graphs of bounded twin-width

Jakub Gajarský, Michał Pilipczuk, Szymon Toruńczyk

University of Warsaw

Twin-width is a graph parameter introduced by Bonnet, Kim, Thomassé and Watrigant [1] in 2020. During the short time since its introduction, it has attracted considerable attention, and its various structural, algorithmic, combinatorial and model theoretic properties have been established.

Intuitively, a graph has twin-width d if it can be constructed by merging larger and larger parts (starting from parts being single vertices) so that at any moment during the construction, every part has a non-trivial interaction with at most d other parts (trivial interaction between two parts means that either no edges, or all edges span across the two parts).

In our work we consider classes of graphs of bounded twin-width from the perspective of monadic stability. Class $\mathcal{C}$ of graphs is monadically stable if there is a bound on the size of largest half-graph contained in any $G \in \mathcal{C}$ as a semi-induced subgraph. We prove that every class of graphs $\mathcal{C}$ that is monadically stable and has bounded twin-width can obtained from some class with bounded sparse twin-width (a class of graphs of bounded twin-width which also has bounded expansion) by a first-order transduction. This generalizes analogous results for classes of bounded linear clique-width [2] and of bounded clique-width [3]. It also implies that monadically stable classes of bounded twin-width are linearly χ-bounded.

Reference

[1] É Bonnet, E. Kim, S. Thomassé, R. Watrigant. Twin-width I: Tractable FO Model Checking, J. ACM (2022).

[2] J. Nešetřil, P. Ossona de Mendez, R. Rabinovich, S. Siebertz. Classes of graphs with low complexity: The case of classes with bounded linear rankwidth, Eur. J. Comb., 2021.

[3] J. Nešetřil, P. Ossona de Mendez, R. Rabinovich, S. Siebertz. Rankwidth meets stability, Symposium on Discrete Algorithms (SODA) 2021.

Sum Labelling Graphs of Maximum Degree two

Henning Fernau, Kshitij Gajjar

National University of Singapore, Singapore

There is a vast body of literature on graph labelling, as testified by a dynamic survey on this topic maintained by Gallian [1]. The 553-page survey mentions over 3000 papers on different ways of labelling graphs. We study one such type of graph labelling that was introduced in 1990 by Harary [2], known as *sum labelling*.

A graph is called a *sum graph* if its vertices can be labelled by distinct positive integers such that there is an edge between two vertices if and only if the sum of their labels is the label of another vertex of the graph. Every graph can be transformed into a sum graph by adding some isolated vertices to the graph; the minimum number of isolated vertices needed for this is known as the *sum number* of the graph.

The sum number of most prominent graph classes (cycles, trees, complete graphs, etc.) is already known. However, beyond these graphs, it is extremely difficult to pin down the sum number of even the most simplistic graph classes. In this work, we examine the effect of taking the disjoint union of graphs on the sum number. In particular, we provide an almost complete characterization of the sum number of graphs of maximum degree two, as every such graph is the disjoint union of paths and cycles.

Reference

[1] J. A. Gallian. A dynamic survey of graph labeling, version 23. *The Electronic Journal of Combinatorics*, DS 6, 2020.

[2] Frank Harary. Sum graphs and difference graphs. *Congressus Numerantium*, 72:101–108, 1990.

Product structure of graph classes with bounded treewidth

Rutger Campbell, Katie Clinch, Marc Distel, J. Pascal Gollin,
Kevin Hendrey, Robert Hickingbotham, Tony Huynh,
Freddie Illingworth, Youri Tamitegama, Jane Tan, David R. Wood

Institute for Basic Science, Daejeon, South Korea

We show that many graphs with bounded treewidth can be described as subgraphs of the strong product of a graph with smaller treewidth and a bounded-size complete graph. To this end, we define the *c-tree-partition-width* of a graph G as the smallest integer w for which G is isomorphic to a subgraph of $H \boxtimes K_w$ where the treewidth of H is at most c, and we define the *underlying treewidth* of a graph class $\mathcal{G}$ to be the minimum non-negative integer c such that, for some function f, for every graph $G \in \mathcal{G}$ has c-tree-partition-width at most $f(\text{tw}(G))$. We introduce a notion of *disjointed partitions* to characterise when a graph of treewidth k has bounded c-tree-partition-width and use this concept to compute the underlying treewidth of several graph classes of interest. As an example, we show that the class of planar graphs has underlying treewidth 3.

The Hamilton compression of highly symmetric graphs

Petr Gregor, Arturo Merino, Torsten Mütze

Charles University, Prague

We say that a Hamilton cycle $C = (x_1, \ldots, x_n)$ in a graph G is k-symmetric, if the mapping $x_i \mapsto x_{i+n/k}$ for all $i = 1, \ldots, n$, where indices are considered modulo n, is an automorphism of G. In other words, if we lay out the vertices $x_1, \ldots, x_n$ equidistantly on a circle and draw the edges of G as straight lines, then the drawing of G has k-fold rotational symmetry, i.e., all information about the graph is compressed into a $360°/k$ wedge of the drawing. We refer to the maximum k for which there exists a k-symmetric Hamilton cycle in G as the *Hamilton compression of* G. We investigate the Hamilton compression of four different families of vertex-transitive graphs, namely hypercubes, Johnson graphs, permutahedra and Cayley graphs of abelian groups. In several cases we determine their Hamilton compression exactly, and in other cases we provide close lower and upper bounds. The cycles we construct have a much higher compression than several classical Gray codes known from the literature. Our constructions also yield Gray codes for bitstrings, combinations and permutations that have few tracks and/or that are balanced.

On multichromatic numbers of widely colorable graphs

Anna Gujgiczer, Gábor Simonyi

Department of Computer Science and Information Theory,
Budapest University of Technology and Economics
and
MTA-BME Lendület Arithmetic Combinatorics Research Group, ELKH, Budapest

A coloring of a graph is called s-wide if no walk of length $2s-1$ connects vertices of the same color. A graph is s-widely colorable with t colors if and only if it admits a homomorphism into a universal graph $W(s,t)$. The talk is about the multichromatic numbers of these universal graphs. We determine the rth multichromatic number of $W(s,t)$ whenever $r \leq s$. The resulting value matches a lower bound proved by Tardif in [2] and answers a question he asked in the same paper. We also discuss our (lack of) knowledge in the $r > s$ case.

The talk is based on the paper [1].

Reference

[1] Anna Gujgiczer and Gábor Simonyi, On multichromatic numbers of widely colorable graphs, *J. Graph Theory*, 100 (2022), 346-361.

[2] Claude Tardif, The chromatic number of the product of 14-chromatic graphs can be 13, *Combinatorica*, 42 (2022), 301-308.

Oriented expressions of graph properties

Santiago Guzmán-Pro, César Hernández-Cruz

Facultad de Ciencias, UNAM, Mexico City

The Roy-Gallai-Hasse-Vitaver Theorem asserts that a graph G is k-colourable if and only if it admits and orientation with no directed walk on $k+1$ vertices. This implies that if F_k is the set of homomorphic images of the directed path on $k+1$ vertices, then a graph is k-colourable if and only if it admits an F_k-free orientation, that is, an orientation with no induced oriented subgraph in F_k. We say that a class of graphs $\mathcal{P}$ is *expressible by forbidden orientations* if there is a finite set F of oriented graphs such that $\mathcal{P}$ is the class of graphs that admit an F-free orientation. Skrien [3] shows that some of these classes include proper circular arc graphs, nested interval graphs and the so-called perfectly orientable graphs. We are interested in the following motivating question: Which graph classes are expressible by forbidden orientations? There are two ways to tackle this question. The first one is to fix a finite set F of oriented graphs and characterize the class of graphs that admit an F-free orientation; we do so in [1] by considering sets of oriented graphs on three vertices. Secondly, we can fix a class of graphs $\mathcal{P}$ and find a finite set of oriented graphs F such that $\mathcal{P}$ is the class of graphs that admit an F-free orientation; we do so in [2] where we show that for every odd cycle C the class of C-colourable graphs is expressible by forbidden orientations. In this talk we present some limitations of this expression system. We do so by exhibiting some necessary conditions upon certain graph classes to be expressible by forbidden orientations. Consequently, we exhibit an uncountable family of hereditary classes for which no such finite set exists. In particular, we show that the class of even hole-free graphs is not expressible by forbidden orientations.

Reference

[1] S. Guzmán-Pro, and C. Hernández-Cruz, Orientations without forbidden patterns on three vertices, arxiv:2003.05606

[2] S. Guzmán-Pro, and C. Hernández-Cruz, Duality pairs and homomorphisms to oriented and unoriented cycles, Electronic Journal of Combinatorics 28(3) (2021) P3.17

[3] D. J. Skrien, A relationship between triangulated graphs, comparability graphs, proper interval graphs, proper circular-arc graphs, and nested interval graphs, Journal of graph Theory 6 (1982) 309–316.

Extremal bipartite biregular bi-coset graphs

Štefan Gyürki, Robert Jajcay, Pavol Jánoš,
Martin Mačaj, Jozef Širáň, Yan Wang

Slovak University of Technology, Bratislava

Let G be a finite group and H, K be its subgroups such that $H \cap K = \{1\}$. The *bi-coset graph* $\Gamma_{(G;H,K)}$ is the bipartite graph whose vertices are the left cosets of H and K in G, and the adjacency relation is defined via non-empty intersection.

Biregular $(m, n; g)$*-graphs* are graphs that consist of vertices of two degrees, m and n (with both of the degrees present) and which are of girth g. *Bipartite biregular* $(m, n; g)$*-graphs* are bipartite $(m, n; g)$-graphs with the additional property that the two partite sets consist of vertices of the same degree (different for each set). For a given triple $(m, n; g)$ a smallest (with respect to the order) bipartite biregular $(m, n; g)$-graph is called a *bipartite biregular* $(m, n; g)$*-cage.* In the talk we present a construction of bipartite biregular $(3, 3k; 6)$-cages which are bi-coset graphs and work when k is odd and $6k + 1$ is a prime.

An edge-girth-regular egr(v, k, g, λ)-graph Γ is a k-regular graph of order v and girth g in which every edge is contained in λ distinct g-cycles. The smallest graphs among all edge-girth-regular graphs for given parameters (k, g, λ) are called *extremal.* Similarly, the smallest bipartite graph among all edge-girth-regular graphs for given parameters (k, g, λ) is called *extremal bipartite edge-girth-regular.* A few infinite families of extremal bipartite edge-girth-regular graphs will be presented that were obtained as bi-coset graphs of matrix groups.

Acknowledgement. The author acknowledges support from the APVV Research Grants 17-0428 and 19-0308, and from the VEGA Research Grants 1/0206/20 and 1/0567/22.

Twin-width of Planar Graphs is at most 11

Petr Hliněný

Faculty of Informatics, Masaryk University, Brno

Twin-width is a new and very successful structural width measure of graphs, which can be seen as "measuring the distance from a cograph". Essentially, it measures how similar are neighbourhoods of the vertices in a graph; by recursively identifying (contracting) similar pairs of vertices and marking differences ("errors") in their neighbourhood by red edges, while keeping the maximum red degree low at all times.

Already the first paper on twin-width by Bonnet et al. [FOCS 2020] included an asymptotic argument bounding the twin-width of planar graphs by a non-explicit constant. Quite recently, we have seen first small explicit upper bounds of 183 by Jacob and Pilipczuk [arXiv:2201.09749, January 2022] (to appear at WG 2022), 583 by Bonnet et al. [arXiv:2202.11858, February 2022], and of 37 by Bekos et al. [arXiv:2204.11495, April 2022].

We prove that the twin-width of planar graphs is at most 11; which means that every planar graph can be brought down to a single vertex by successively identifying pairs of vertices and never having the red degree more than 11. Unlike the previously mentioned results, our proof technique is not directly related to the product structure of planar graphs, but still inspired by the former approaches. We also expect further slight improvement of this bound in the near future. Details can be found in [arXiv:2205.05378].

Gadget construction for elementary convergence of relational structures

David Hartman, Tomáš Hons, Jaroslav Nešetřil

Computer Science Institute of Charles University, Prague

There are multiple ways to define convergence of graphs and corresponding graph limits. One of them, called *structural convergence*, proposed by Nešetřil and Ossona de Mendez [1], defines convergence of relational structures using logical formulas in a way generalizing some other approaches to graph convergence, such as *dense graph limits* and *Benjamini-Schramm convergence*. By restricting to sentences, we get a case of structural convergence called *elementary convergence*: A sequence of relational structures is elementary convergent if for each first order sentence there exists an index from which on all the structures model the sentence or none of them does.

We consider the gadget (indicator) construction (see e.g. [2]) applied entry-wise on a series of structures and systems of series of gadgets. Our interest lies in the case when (some of) these sequences converge. One of our main results can be described as follows.

For an elementary convergent series of structures and an elementary convergent system of gadgets the resulting series created using the gadget construction is elementary convergent.

This construction enables us to generate various examples of convergent series. In addition, we can describe the limit object of sequences created by this method as a countable relational structure. This together with other results will appear in [3].

This work is part of a project that has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (grant agreement No 810115 – Dynasnet).

Reference

[1] Nešetřil, J., Ossona de Mendez, P. A Unified Approach to Structural Limits and Limits of Graphs with Bounded Tree-Depth. *Mem. Am. Math. Soc.*. **263**(1272), 2020.

[2] Hell, P., Nešetřil, J. *Graphs and homomorphisms.* OUP Oxford, 2004.

[3] Hartman, D., Hons, T., Nešetřil, J. Gadget construction for elementary convergence of relational structures. *in preparation.*

On the maximum number of perfect matchings

Peter Horak

University of Washington

This presentation will consists of two parts. The first part will discuss a joint result with Dongryul Kim, a graduate student at Stanford; the second part is devoted to my reminiscences of Robin Thomas.

In 2008 Alon and Friedland showed that a simple cubic graph G on $2n$ vertices has at most $6^{n/3}$ perfect matchings, and this bound is attained by taking the disjoint union of bipartite complete graphs $K_{3,3}$. In other words, the above theorem says that the complete bipartite graph $K_{3,3}$ has the highest "density" of perfect matchings among all cubic graphs. However, this result does not provide any insight into the structure of extremal connected cubic graphs. The main result of this part claims that, for $n \geq 6$, the number of perfect matchings in a simple connected cubic graph on $2n$ vertices is at most $4f_{n-1}$, with f_n being the n-th Fibonacci number, and a unique extremal graph will be characterized as well.

Robin Thomas has proved seminal and acclaimed results in a wide range of mathematical fields. In my reminiscences of Robin, I recall a one that is probably not as well known. Robin proved this math gem when he was still an undergraduate, and likely it was his very first result.

Rainbow bases in matroids

Florian Hörsch, Tomáš Kaiser, Matthias Kriesell

Technische Universität Ilmenau, Germany

Recently, it was proved by Bérczi and Schwarcz that the problem of factorizing a matroid into rainbow bases with respect to a given partition of its ground set is algorithmically intractable, leaving many special cases open.

We first show that the problem remains hard if the matroid is graphic, answering a question of Bérczi and Schwarcz. Solving another special case, we show that the problem of deciding whether a given digraph can be factorized into subgraphs which are spanning trees in the underlying sense and respect upper bounds on the indegree of every vertex is also hard. This answers a question of Frank.

Further, we deal with the relaxed problem of covering the ground set of a matroid by rainbow bases. Among other results, we show that there is a linear function f such that every matroid that can be factorized into k bases for some $k \geq 3$ can be can be covered by $f(k)$ rainbow bases if every partition class contains at most 2 elements.

Extension property for partial automorphisms: cobinatorial constructions and open problems

Jan Hubička, Matěj Konečný, Jaroslav Nešetřil

Department of Applied Mathematics (KAM), Charles University, Prague

In 1992 Hrushovski shown that for every finite graph G there exists finite graph H containing G as an induced subgraph such that every isomorphism of two induced subgraphs of G extends to an automorphism of H [3]. This property (of the class of finite graphs) is called *the Extension Property for Partial Automorphisms* (*EPPA*) or *the Hrushovski property* and has several implications to the properties of the automorphism group of the countable random graph (Rado graph). In fact, it was formulated in order to solve the (group-theoretic) small index property conjecture [2].

Hrushovski's proof uses group-theoretic tools. An elementary combinatorial proof, based on the intersection graphs, was given by Herwig and Lascar [2]. In the same paper they proved the Herwig and Lascar theorem (one of deepest results in the area), which gives a structural condition for a class of structures to have EPPA. This gives many additional classes with EPPA and is again proved using group theoretic methods.

We show different elementary combinatorial construction which generalizes to new proof of the Herwig-Lascar theorem and its strengthening for classes of structures with unary functions [4].

Reference

[1] B. Herwig, D. Lascar, Extending partial automorphisms and the profinite topology on free groups, Transactions of the American Mathematical Society 5 (2000), 985–2021

[2] W. Hodges, I. Hodkinson, D. Lascar, S. Shelah, The small index property for ω-stable ω-categorical structures and for the random graph, Journal of the London Mathematical Society 2 (1993): 204–218.

[3] E. Hrushovski, Extending partial isomorphisms of graphs, Combinatorica 4 (1992), 411–416.

[4] J. Hubička, M. Konečný, J. Nešetřil, All those EPPA classes (Strengthenings of the Herwig-Lascar theorem), arXiv:1902.03855, Transactions of the American Mathematical Society, in print, 67 pages.

Counting Circuit Double Covers

Radek Hušek, Robert Šámal

*Faculty of Information Technology, Czech Technical University in Prague**

We study a counting version of Cycle Double Conjecture:

Conjecture 1 (Szekeres '73 [1]). *Every bridgeless graph has a circuit double cover.*

We count circuit double covers[1] instead of k-cycle double covers because the number of k-CDC heavily depends on k. We show almost exponential lower bound for graphs with a nice embedding on surfaces and exponential lower bound for planar graphs:

Theorem 2. *Every bridgeless cubic graph with embedding into a surface of Euler characteristic χ with representativity at least 4 has $2^{\Omega(\sqrt[3]{n+2\chi})}$ circuit double covers.*

Theorem 3. *Every bridgeless cubic planar graph has at least $(5/2)^{n/4-1/2}$ circuit double covers.*

Based on conducted experiments, we also present a strengthening of CDC conjecture:

Conjecture 4. *Every bridgeless cubic graphs with n vertices has at least $2^{n/2-1}$ circuit double covers.*

We know that the hypothetical minimal counterexample is cyclically 4-edge-connected, does not contain C_4 and has at least 22 vertices. This conjecture is tight for graphs obtained from K_4 by expanding vertices to triangles. On the other hand, it might not be the strongest possible for cyclically 4-edge-connected graphs.

Reference

[1] George Szekeres. Polyhedral decompositions of cubic graphs. *Bulletin of the Australian Mathematical Society*, 8(3):367–387, 1973. doi: 10.1017/S0004972700042660.

*Large part of the work was done during my Ph.D. study at Faculty of Mathematics and Physics, Charles University, Prague.

[1] Circuit is a graph isomorphic to C_k for some k. Cycle is graphs with all degrees even (i.e., edge-disjoint union of circuits).

A connection between G-graphs and lifting construction

Štefan Gyürki, Pavol Jánoš, Jana Šiagiová, Jozef Širáň

Slovak University of Technology, Bratislava

The problem of finding (k, g)-cages is to find the smallest (in terms of the number of vertices) k-regular graphs of girth g. One of the approaches of finding small k-regular graphs of given girth are constructions based on groups; a prominent example are lifting constructions, which can be regarded as a generalisation of the well known Cayley graphs. Bretto and Faisant presented in [1] another construction of graphs related to groups and having highly regular properties, called G-graphs.

In the talk we compare these two constructions and derive a sufficient condition providing when the G-graphs can be obtained as lifts of dipoles. We also provide the lifting constructions of near-cages of girth 6 and 8, originally constructed in [2] as p-regular G-graphs for an arbitrary prime p, which we were able to extend for prime powers.

The second author acknowledges support from the APVV Research Grants 17-0428 and 19-0308, and from the VEGA Research Grants 1/0206/20 and 1/0567/22.

Reference

[1] A. Bretto and A. Faisant, A new way for associating a graph to a group, Math. Slovaca **55**(1) (2005), 1–8.

[2] A. Bretto, A. Faisant and L. Gillibert, New graphs related to $(p, 6)$ and $(p, 8)$-cages, Comput. Math. Appl. **62** (2011), 2472–2479.

Twin-width and Transductions of Proper k-Mixed-Thin Graphs

Jakub Balabán, Petr Hliněný, Jan Jedelský

Masaryk University, Brno

Twin-width is a graph parameter recently introduced by Bonnet, Kim, Thomassé and Watrigant [1]. Since classes of bounded twin-width admit fixed-point tractable first-order model checking given a suitable vertex ordering, it is interesting to study them.

One of the natural classes of graphs of bounded twin-width are proper interval graphs, a class generalized to proper k-mixed-thin graphs by Bonomo and Estrada [2]. We generalize this class even further and define proper k-mixed-thin graphs.

We show that the twin-width of proper k-mixed-thin graphs is linear in k (and give the appropriate vertex ordering), using an approach called red-potential which was developed by Balabán and Hliněný [3] to improve bound on twin-width of posets of bounded width.

Boundedness of twin-width is preserved by transductions, making the transduction hierarchy of bounded twin-width classes also interesting. We show that posets of bounded width are transduction equivalent to a subclass of proper k-mixed-thin graphs.

Reference

[1] Bonnet, É., Kim, E.J., Thomassé, S., Watrigant, R.: Twin-width I: tractable FO model checking. In: FOCS. pp. 601–612. IEEE (2020)

[2] Bonomo, F., de Estrada, D.: On the thinness and proper thinness of a graph. Discret. Appl. Math. **261**, 78–92 (2019)

[3] Balabán, J., Hliněný, P.: Twin-width is linear in the poset width. In: IPEC. LIPIcs, vol. 214, pp. 6:1–6:13. Schloss Dagstuhl - Leibniz-Zentrum für Informatik (2021)

List homomorphism problems for signed graphs

Jan Bok, Richard Brewster, Tomás Feder, Pavol Hell,
Nikola Jedličková, and Arash Rafiey

Department of Applied Mathematics, Faculty of Mathematics and Physics, Charles University, Prague

We consider homomorphisms of signed graphs from a computational perspective. In particular, we study the list homomorphism problem seeking a homomorphism of an input signed graph (G, σ), equipped with lists $L(v) \subseteq V(H), v \in V(G)$, of allowed images, to a fixed target signed graph (H, π). The complexity of the similar homomorphism problem without lists (corresponding to all lists being $L(v) = V(H)$) has been previously classified by Brewster and Siggers, but the list version remains open and appears difficult. We will summarise the results towards a dichotomy. First, we classify the complexity of the problem when H is a tree (with possible loops). Kim and Siggers have conjectured a structural classification in the special case of the so called weakly balanced signed graphs, and proved it for reflexive signed graphs. We confirm the conjecture for irreflexive signed graphs; this generalizes previous results on weakly balanced signed trees, and weakly balanced separable signed graphs.

Hamilton cycles in line graphs of hypergraphs

Tomáš Kaiser, Petr Vrána

University of West Bohemia, Plzeň

Interest in Hamilton cycles in line graphs has been largely motivated by the conjecture of Thomassen [3] which states that all 4-connected line graphs are Hamiltonian. While the conjecture remains open, it is known to hold with 4 replaced by 6 (cf. [2]). In contrast, no similar result is known for line graphs of hypergraphs of rank $r \geq 3$.

In this talk, we outline a first step in this direction and show that 52-connected line graphs of rank 3 hypergraphs are Hamiltonian. The proof uses a result on the packing of T-connectors in graphs due to DeVos et al. [1] (see also [4]). Joint work with Petr Vrána.

Reference

[1] M. DeVos, J. McDonald and I. Pivotto, Packing Steiner trees, J. Combin. Theory Ser. B 119 (2016), 178–213.

[2] T. Kaiser and P. Vrána, Hamilton cycles in 5-connected line graphs, European J. Combin. 33 (2012), 924–947.

[3] C. Thomassen, Reflections on graph theory, J. Graph Theory 10 (1986), 309–324.

[4] D. B. West and H. Wu, Packing of Steiner trees and S-connectors in graphs, J. Combin. Theory Ser. B 102 (2012), 186–205.

Metric and non-metric hypergraphs

Vašek Chvátal, Ida Kantor

IÚUK, Charles University, Prague

In a metric space $M = (X, d)$, we say that b *is between* a *and* c if $d(a, c) = d(a, b)+d(b, c)$. Taking all triples $\{a, b, c\}$ such that a is between b and c, one can associate a 3-uniform hypergraph $\mathcal{H}_M$ with each finite metric space M. An effort to solve some basic open questions regrading finite metric spaces has motivated an endeavor to better understand these associated hypergraphs. We present results in this direction, most notably some simple families of hypergraphs that are non-metric, i.e., they don't arise from any metric space.

Minimally t-tough graphs in special graph classes

Gyula Y. Katona

Budapest University of Technology and Economics, Budapest, Hungary

A graph G is minimally t-tough if the toughness of G is t and the deletion of any edge from G decreases the toughness. Kriesell conjectured that for every minimally 1-tough graph the minimum degree $\delta(G) = 2$. It is natural to generalize this for other t values: Every minimally t-tough graph has a vertex of degree $\lceil 2t \rceil$.

In the present talk we investigate different questions related to this conjecture.

The conjecture seems to be hard to prove, so we tried to prove it for some special graph classes. It turned out, that in some cases the conjecture is true because there are very few or no graphs that satisfy the conditions. On the other hand, we have evidence using complexity theory, that this is not the situation for some other graph classes.

We investigate the minimum degree and the recognizability of minimally t-tough graphs in the classes of chordal graphs, split graphs, claw-free graphs, and $2K_2$-free graphs.

One of the most interesting results is that there is no minimally t-tough strongly chordal graph for $t > 1/2$. This is proved by a powerful necessary and sufficient condition we proved for a graph being minimally t-tough. For $t \leq 1/2$ on the other hand there exists minimally t-tough chordal graphs, moreover we can characterize them, they have very special structure.

We conjecture that there is no minimally t-tough chordal graph for $t > 1/2$. Many other open question remain.

All results are joint work with various subsets of the following coauthors: C. Dallard, B. Fernández, H. Khan, M. Milanič, K. Varga.

Minimally tough chordal graph with small toughness

Gyula Y. Katona, Humara Khan

Budapest University of Technology and Economics, Budapest, Hungary

Let t be a real number. A graph is called *t-tough* if the removal of any vertex set S that disconnects the graph leaves at most $|S|/t$ components. The toughness of a graph is the largest t for which the graph is t-tough. A graph is minimally t-tough if the toughness of the graph is t and the deletion of any edge from the graph decreases the toughness. A graph is *chordal* if it does not contain an induced cycle of length at least 4. We will call a graph a *TT-graph* if it can be obtained from a tree of maximum degree $\Delta > 3$ by removing some (or all) of its vertices with degree 3 whose neighbors have degree Δ, and joining these neighbors with triangle. For $\Delta \leq 3$ the definition is similar, but slightly more complicated.

Kriesell's conjectured [1] that every minimally 1-tough graph has a vertex of degree 2. This conjecture can be naturally generalized: every minimally t-tough graph has a vertex of degree $\lceil 2t \rceil$. Gyula Y. Katona and Kitti Varga [3], showed that the conjecture is true for chordal graphs when $1/2 < t \leq 1$.

In this paper we show that the Generalized Kriesell's Conjecture for chordal graphs with toughness $\leq 1/2$ by giving a characterization of such graphs. We show that for $t \leq 1/2$ a chordal graph is minimally t-tough if and only if it is a TT-graph. As a corollary, a characterization of minimally t-tough interval graphs is obtained for $t \leq 1/2$, as well.

Reference

[1] Mathias Kriesell, *In: Ed. T. Kaiser, Problems from the Workshop on dominating cycles*, Hájek, Czech Republic, 2003.
`http://iti.zcu.cz/history/2003/Hajek/problems/hajek-problems.ps`

[2] C. G. Lekkerkerker and J. Ch. Boland, *Representation of a finite graph by a set of intervals on the real line*, Fund. Math., 51 (1962), pp. 45–64.

[3] Gyula Y. Katona, Kitti Varga, *Minimally toughness in special graph classes,* arXiv:1802.00055 [math.CO] 2018.

Domination in regular graphs

Martin Knor, Riste Škrekovski, Aleksandra Tepeh

Slovak University of Technology in Bratislava, Bratislava

Let G be a graph. A set $S \subseteq V(G)$ is *dominating* if $N[S] = V(G)$. If S is also independent, then it is an *independent dominating set. Dominating number* $\gamma(G)$ (*independent dominating number* $i(G)$) is the minimum cardinality of an (independent) dominating set in G.

Babikir and Henning conjectured that if G is a k-regular graph then

$$\frac{i(G)}{\gamma(G)} \leq \frac{k}{2}$$

with equality if and only if $G = K_{k,k}$. We proved this conjecture.

This research was partially supported by Slovak research grants APVV-17-0428, APVV-19-0308, VEGA 1/0206/20 and VEGA 1/0567/22.

Grid Induced Minor Theorem for Graphs of Small Degree

Tuukka Korhonen

University of Bergen, Norway

A graph H is an induced minor of a graph G if H can be obtained from G by vertex deletions and edge contractions. We show that there is a function $f(k, d) = \mathcal{O}(k^{10} + 2^{d^5})$ so that if a graph has treewidth at least $f(k, d)$ and maximum degree at most d, then it contains a $k \times k$-grid as an induced minor. This proves the conjecture of Aboulker, Adler, Kim, Sintiari, and Trotignon [1] that any graph with large treewidth and bounded maximum degree contains a large wall or the line graph of a large wall as an induced subgraph. It also implies that for any fixed planar graph H, there is a subexponential time algorithm for maximum weight independent set on H-induced-minor-free graphs.

The full version of this paper is available at [2].

Reference

[1] Pierre Aboulker, Isolde Adler, Eun Jung Kim, Ni Luh Dewi Sintiari, and Nicolas Trotignon. On the tree-width of even-hole-free graphs. *Eur. J. Comb.*, 98:103394, 2021.

[2] Tuukka Korhonen. Grid Induced Minor Theorem for Graphs of Small Degree. *arXiv preprint*, arXiv:2203.13233, 2022.

Vertex in-out-antimagic total labeling of digraphs

Martin Bača, Petr Kovář, Tereza Kovářová, Andrea Semaničová-Feňovčíková

VSB – Technical University of Ostrava, Ostrava

A vertex in-out-antimagic total labeling of a directed graph (digraph) $D = (V, A)$ with n vertices and m arcs is a bijection from the set of vertices and edges to the set of the first $m + n$ integers such that all n vertex in-weights are pairwise distinct and simultaneously all n vertex out-weights are pairwise distinct, where the vertex in-weight is the sum of the vertex label and the labels of all incoming arcs and the vertex out-weight is the sum of the vertex label and the labels of all outgoing arcs.

It was conjectured that all digraphs allow such labeling. A general way how to label dense digraphs was provided. The real challenge is in labeling sparse digraphs. We provide some new results concerning balanced digraphs and regular digraphs.

Recent Progress in the Computational Complexity of Graph Covers

Jan Bok, Jiří Fiala, Nikola Jedličková, Jan Kratochvíl, Michaela Seifrtová

Charles University, Prague

The notion of *graph covers* stems from topological graph theory, but has found its applications in Computer Science in the theory of local computation. In the realm of simple connected undirected graphs this notion coincides with the notion of a *locally bijective homomorphism*, i.e., an adjacency preserving vertex mapping which is a bijection between the neighborhoods of a vertex and of its image, for every vertex of the source graph. Modern topological graph theory and mathematical physics prefer to work with multigraphs with loops and also so-called semi-edges (a semi-edge is an edge incident to a single vertex only, but contributing only 1 to the degree of this vertex, while a loop contributes 2). In case of such generally defined graphs, a graph covering projection from a graph G to a graphs H is a pair of mappings $(f_v : V(G) \longrightarrow V(H), f_e : E(H) \longrightarrow E(H))$ such that for any two vertices $x, y \in V(H)$,

- a loop incident with a vertex $x \in V(H)$ is mapped onto a loop incident with $f_v(x)$;
- a semi-edge incident with a vertex $x \in V(H)$ is mapped onto a semi-edge incident with $f_v(x)$;
- a normal edge incident with two distinct vertices $x, y \in V(H)$ is mapped onto a normal edge incident with the vertices $f_v(x), f_v(y)$ (if $f_v(x) \neq f_v(y)$) or onto a loop or semi-edge incident with $f_v(x) = f_v(y)$;
- for every vertex $x \in V(G)$ and every normal edge or semi-edge $\beta \in E(H)$, there exists exactly one edge α of G such that $f_e(\alpha) = \beta$; and
- for every vertex $x \in V(G)$ and every loop $\beta \in E(H)$, there exist exactly two edges of G that are mapped onto β by f_e.

In 1991, Abello et al. initiated the quest for cataloging the complexity of deciding if an input graph G covers a target graph H, parameterized by the target graph H. Taking the semi-edges into account is enriching and dramatically changing the (so far very incomplete) catalog. We will survey the most recent results in this area, including a complete characterization for graphs H with at most two vertices per equivalence class in the degree partition of the graph, or the list-covering version of the problem.

It is worth noting that all cases in which the computational complexity is known to us, the question is either polynomial time solvable for arbitrary inputs, or it is NP-complete for simple input graphs. This phenomenon has been conjectured to hold true for all fixed target graphs in one of our recent papers, and we refer to it as the *Strong Dichotomy Conjecture for Graph Covers.*

Aspects of Kasteleyn Orientations

Martin Loebl

Charles University, Prague

The theory of Kasteleyn orientation is a fundamental computational and strategic tool for theoretical physics, discrete mathematics and computer science. This theory was one of the favourite topics of Robin Thomas. In my lecture, I will recall the history and then point out parts of the theory perhaps less known to the graph theory community such as the determinantal complexity, holographic algorithms and the discrete Ihara-Selberg function.

Circular flows and balanced valuations

Robert Lukoťka

Comenius University, Bratislava

An r-balanced flow is an assignment b of values to the vertices of G, and an orientation O and assignment ϕ of values to the edges of G such that

- $b(v) \equiv \deg(v)$, for each vertex v,
- $0 \leq \phi(e) \leq \frac{r-2}{r}$, for each edge e,
- $\sum_{e \in O^+} \phi(e) - \sum_{e \in O^+} \phi(e) = b(v)$, for each vertex v.

We show that this concept coincides with the concept of balanced valuations introduced by Jaeger [1] and thus it coincides with circular nowhere-zero r-flows. This definition gives an practical approach to compute circular flow number graphs on up to approximately 100 vertices using state of the art ILP solvers. We discuss how similar ideas could be used for dual concept of circular colourings.

Reference

[1] F. Jaeger, Balanced valuations and flows in multigraphs, Proceedings of the American Mathematical Society 55 (1976), 237–242.

Robust Connectivity of Graphs on Surfaces

Peter Bradshaw, Tomáš Masařík, Jana Novotná, Ladislav Stacho

University of Warsaw, Warsaw, Poland

Let $\Lambda(T)$ denote the set of leaves in a tree T. One natural problem is to look for a spanning tree T of a given graph G such that $\Lambda(T)$ is as large as possible. This problem is called MAXIMUM LEAF NUMBER, and it is a well-known NP-hard problem. Equivalently, the same problem can be formulated as the MINIMUM CONNECTED DOMINATING SET problem, where the task is to find a smallest subset of vertices $D \subseteq V(G)$ such that every vertex of G is in the closed neighborhood of D. Throughout recent decades, these two equivalent problems have received considerable attention, ranging from pure graph theoretic questions to practical problems related to the construction of wireless networks.

Recently, a similar but stronger notion was defined by Bradshaw, Masařík, and Stacho [2]. They introduced a new invariant for a graph G, called the *robust connectivity* and written $\kappa_\rho(G)$, defined as the minimum value $\frac{|R\cap\Lambda(T)|}{|R|}$ taken over all nonempty subsets $R \subseteq V(G)$, where $T = T(R)$ is a spanning tree on G chosen to maximize $|R \cap \Lambda(T)|$. Large robust connectivity was originally used to show flexible choosability in non-regular graphs.

In this paper, we investigate some interesting properties of robust connectivity for graphs embedded in surfaces. We prove a tight asymptotic bound of $\Omega(\gamma^{-\frac{1}{r}})$ for the robust connectivity of r-connected graphs of Euler genus γ. Moreover, we give a surprising connection between the robust connectivity of graphs with an edge-maximal embedding in a surface and the *surface connectivity* of that surface, which describes to what extent large induced subgraphs of embedded graphs can be cut out from the surface without splitting the surface into multiple parts. For planar graphs, this connection provides an equivalent formulation of a long-standing conjecture of Albertson and Berman [1], which states that every planar graph on n vertices contains an induced forest of size at least $n/2$.

Reference

[1] Michael O. Albertson and David M. Berman, A conjecture on planar graphs, Graph theory and related topics 357 (1979), 1.

[2] Peter Bradshaw, Tomáš Masařík, and Ladislav Stacho, Flexible list colorings in graphs with special degeneracy conditions, accepted in Journal of Graph Theory (2022+).

Cut-through-paths in simple 4-regular plane graphs

Tomáš Madaras, Daniela Matisová, Juraj Valiska

Pavol Jozef Šafárik University, Košice

Let G be a 4-regular graph with prescribed rotation system and let e_1, e_2, e_3, e_4 be edges incident with a vertex v in that order. The pairs e_1, e_3 and e_2, e_4 are called CT-adjacent in G. A CT-path (CT-trail) is a path (trail) in which every two consecutive edges are CT-adjacent. Simple 4-regular plane graphs consisting of a single closed CT-trail are called knots; if every closed CT-trail of a simple 4-regular plane graph is a CT-cycle, then the graph is called Grötzsch-Sachs graph.

In this talk, we show that the longest CT-path in an n-vertex knot has at most $n-2$ vertices, and give construction of knot with longest CT-path with that number of vertices for every $n \geq 8$; also we prove that the longest CT-path in an n-vertex Grötzsch-Sachs graph has at most $\frac{2n}{3}$ vertices. Next, we show that there exists infinitely many simple 4-regular plane graphs whose longest CT-paths contain just eight vertices; we conjecture that, apart of the single exception, all graphs with longest 8-vertex paths are Grötzsch-Sachs graphs. In addition, we provide an analogous construction yielding knots with longest 16-vertex paths. In the case when the longest CT-path has less than eight vertices, we pose a conjecture (supported by computer simulations generating the list of feasible graphs) that there is only finitely many corresponding 4-regular plane graphs; we have confirmed its validity for longest CT-paths on four and five vertices.

1-designs and related combinatorial structures and linear codes

Vedrana Mikulić Crnković, Ivona Traunkar

Faculty of Mathematics, University of Rijeka, Rijeka, Croatia

Let G be a transitive group acting on a set Ω, P a subgroup of G and Δ be a union of some P orbits on Ω. Then Δ is the base block of a 1-design. We will apply this known method of construction of 1-designs to construct t-designs, regular graphs, and digraphs on which the group G acts transitively as an automorphism group.

Additionally, a 1-design is weakly self-orthogonal if all the block intersection numbers have the same parity. If both k and the block intersection numbers are even then 1-design is called self-orthogonal and its incidence matrix generates a self-orthogonal code. We analyze extensions of the incidence matrix and an orbit matrix of a weakly self-orthogonal 1-design that generates self-orthogonal or LCD code over an arbitrary field.

Graphs and vertex orders in Discretizable Distance Geometry

Antonio Mucherino

IRISA, University of Rennes 1, Rennes, France

Given a simple weighted undirected graph $G = (V, E, d)$ and a positive integer K, the Distance Geometry Problem (DGP) asks whether a realization $x : V \to \Re^K$ exists such that the distances between embedded vertices u and $v \in V$ coincide with the weights $d_{u,v}$, when available. While the DGP search space is in general continuous, the subclass of DGP instances for which this search space can be reduced to a tree is of particular interest. In such a case, in fact, the complete enumeration of the DGP solution set is potentially possible. We refer to this subclass as the Discretizable DGP (DDGP) [2], and to DGP instances belonging to this class as *discretizable* instances.

Graphs are involved in both the definition of DGP instances (so that the properties of a graph actually allow us to decide whether a DGP instance is discretizable or not), and in the construction of the DDGP search space. In this work, we focus our attention on the latter, and in particular on the possibility to reduce the size of the tree structure (in terms of nodes) when specific vertex orders are associated to the vertex set V [1, 3]. Vertex orders are in fact able to define the dependency of every vertex $v \in V$ with a subset of its precedessors in the given ordering. Recent insights seem to suggest that globally optimal vertex orders can be identified for each DDGP instance, even when the distance information (given through the weight function d) is imprecise.

This work is partially supported by the ANR project ANR-19-CE45-0019.

Reference

[1] D.S. Gonçalves, A. Mucherino, *Optimal Partial Discretization Orders for Discretizable Distance Geometry*, International Transactions in Operational Research **23**(5), 947–967, 2016.

[2] A. Mucherino, C. Lavor, L. Liberti, *The Discretizable Distance Geometry Problem*, Optimization Letters **6**(8), 1671–1686, 2012.

[3] J. Omer, A. Mucherino, *The Referenced Vertex Ordering Problem: Theory, Applications and Solution Methods*, Open Journal of Mathematical Optimization **2**, article no. 6, 29 pages, 2021.

Efficient generation of elimination trees and Hamilton paths on graph associahedra

Jean Cardinal, Arturo Merino, Torsten Mütze

University of Warwick
and
Charles University Prague

An elimination tree for a connected graph G is a rooted tree on the vertices of G obtained by choosing a root x and recursing on the connected components of $G-x$ to produce the subtrees of x. Elimination trees appear in many guises in computer science and discrete mathematics, and they are closely related to centered colorings and tree-depth. They also encode many interesting combinatorial objects, such as bitstrings, permutations and binary trees. We apply the recent Hartung-Hoang-Mütze-Williams combinatorial generation framework (SODA 2020) to elimination trees, and prove that all elimination trees for a chordal graph G can be generated by tree rotations using a simple greedy algorithm. This yields a short proof for the existence of Hamilton paths on graph associahedra of chordal graphs. Graph associahedra are a general class of high-dimensional polytopes introduced by Carr, Devadoss, and Postnikov, whose vertices correspond to elimination trees and whose edges correspond to tree rotations. As special cases of our results, we recover several classical Gray codes for bitstrings, permutations and binary trees, and we obtain a new Gray code for partial permutations. Our algorithm for generating all elimination trees for a chordal graph G can be implemented in time $O(m+n)$ per generated elimination tree, where m and n are the number of edges and vertices of G, respectively. If G is a tree, we improve this to a loopless algorithm running in time $O(1)$ per generated elimination tree. We also prove that our algorithm produces a Hamilton cycle on the graph associahedron of G, rather than just Hamilton path, if the graph G is chordal and 2-connected. Moreover, our algorithm characterizes chordality, i.e., it computes a Hamilton path on the graph associahedron of G if and only if G is chordal. Implementations of these algorithms are available for experimentation on the Combinatorial Object Server: `www.combos.org/elim`

Short chains in poset-free families

József Balogh, Dániel Gerbner, Abhishek Methuku, Dániel T. Nagy,
Balázs Patkós, Ryan R. Martin, Máté Vizer

Alfréd Rényi Institute of Mathematics, Budapest, Hungary

Let $\mathcal{F}$ be a family of sets and P be a partially ordered set (poset). We say that $\mathcal{F}$ is P-free if there is no injective map $f : P \to \mathcal{F}$ such that $f(a) \subset f(b)$ holds for all pairs $a, b \in P$, $a < b$. The problem of finding the largest P-free family formed by subsets of $[n] = \{1, 2, \ldots, n\}$ is solved for certain classes of posets, but open for general P. Inspired by this problem, we investigated a variant of it asking for the largest number of k-chains $A_1 \subset A_2 \subset \cdots \subset A_k$ in a P-free family of subsets of $[n]$.

In [1] we proved bounds on this quantity depending on the length of the longest chain in P. This establishes the solution's order of magnitude for many (P, k) pairs.

In [2] we proved that for all s, t and k, a $K_{s,t}$-free family (a family without sets $A_1, \ldots, A_s, B_1, \ldots, B_k$ such that $A_i \subset B_j$ for all i, j) contains at most $O(n\binom{n}{\lfloor n/2 \rfloor})$ k-chains. The maximal number of 2-chains in a $K_{2,2}$-free family was exactly determined as $\lceil \frac{n}{2} \rceil \binom{n}{\lfloor n/2 \rfloor}$, given by the family of all subsets of size $\lceil \frac{n}{2} \rceil - 1$ and $\lceil \frac{n}{2} \rceil$.

In the talk I will summarize the results of these papers and present a few open questions.

Reference

[1] D. Gerbner, A. Methuku, D.T. Nagy, B. Patkós, M. Vizer, On the number of containments in P-free families, Graphs and Combinatorics 35 (2019), 1519–1540.

[2] J. Balogh, R.R. Martin, D.T. Nagy, B. Patkós, On generalized Turán results in height two posets, SIAM Journal on Discrete Mathematics (accepted)

Decycling 3-connected cubic graphs

Michaela Seifrtová, Roman Nedela, Martin Škoviera

University of West Bohemia, Pilsen

A set of vertices of a graph G is said to be decycling if its removal leaves an acyclic subgraph. The size of a smallest decycling set is the decycling number of G. Generally, at least $\lceil (n+2)/4 \rceil$ vertices have to be removed in order to decycle a cubic graph on n vertices. In 1979, Payan and Sakarovitch proved that the decycling number of a cyclically 4-edge-connected cubic graph of order n equals $\lceil (n+2)/4 \rceil$. In addition, they characterised the structure of minimum decycling sets and their complements. If $n \equiv 2 \pmod 4$, then G has a decycling set which is independent and its complement induces a tree. If $n \equiv 0 \pmod 4$, then one of two possibilities occurs: either G has an independent decycling set whose complement induces a forest of two trees, or the decycling set is near-independent (which means that it induces a single edge) and its complement induces a tree. In this paper we strengthen the result of Payan and Sakarovitch by proving that the latter possibility (a near-independent set and a tree) can always be guaranteed. Moreover, we relax the assumption of cyclic 4-edge-connectivity to significantly weaker odd cyclic 4-edge-connectivity, and even further. Our methods substantially use a surprising and seemingly distant relationship between the decycling number and the maximum genus of a cubic graph. The statement that a cyclically 4-connected cubic graph admits a partition $A \cup J$ of the vertex-set, where J is (near) independent and A induces a tree can be used to prove existence of hamilton cycles or paths for several interesting classes of cubic graphs including leapfrog fullerines or cubic Cayley graphs defined by generating sets $\langle x, y \rangle$, where $y^2 = (xy)^3 = 1$, or by generating sets $\langle a, b, c \rangle$, where $a^2 = b^2 = c^2 = (ab)^3 = (bc)^3 = 1$.

Maximal Independent Sets in Clique-free Graphs

Xiaoyu He, Jiaxi Nie, Sam Spiro

Max Plack Institute for Mathematics in the Sciences, Leipzig

Nielsen proved that the maximum number of maximal independent sets (MIS's) of size k in an n-vertex graph is asymptotic to $(n/k)^k$, with the extremal construction a disjoint union of k cliques with sizes as close to n/k as possible. In this paper we study how many MIS's of size k an n-vertex graph G can have if G does not contain a clique K_t. We prove for all fixed k and t that there exist such graphs with $n^{\lfloor \frac{(t-2)k}{t-1} \rfloor - o(1)}$ MIS's of size k by utilizing recent work of Gowers and B. Janzer on a generalization of the Ruzsa-Szemerédi problem. We prove that this bound is essentially best possible for triangle-free graphs when $k \leq 4$.

Incidence, a scoring positional game on graphs

Quentin Deschamps, Éric Duchêne, Bastien Durain,
Brice Effantin, Valentin Gledel, and Nacim Oijid

Univ. Lyon, Université Lyon 1, LIRIS UMR CNRS 5205, F-69621, Lyon, France

Positional games have been introduces by Erdös and Selfridge in 1973 [1]. Thoses games are playing on a hypergraph: two players select turn by turn an unclaimed vertex of it. In the Maker-Breaker convention, if Maker manages to take fully a hyperedge, he wins, otherwise, Breaker is the winner. In the Maker-Maker convention, the first player to take a hyperedge wins, if no one manage to do it, the game ends by a draw. Therefore, in both cases the game stops as soon as Maker has taken a hyperedge. Theses games do not handle scores, and cannot represent games in which players want to maximize a quantity. We here introduce a scoring version of positional games where the game ends when all vertices have been claimed. The score is then defined as the number of hyperedges Maker manages to take. A Maker player will try to maximize this number, while Breaker aims to minimize it. Usually, Maker-Breaker games are easier to handle than Maker-Maker as in the former, both players can focus only on one goal, while in the later, they need both to try to take an hyperedge and in the same time prevent their opponent to take one.

In this work, we introduce Incidence, which consists in playing this game on a 2-regular hypergraph, i.e. an undirected graph. Two players, namely Alice and Bob, take turn by turn the vertices of a graph, and score the number of edges whose both end vertices are owned by the same player. In the Maker-Breaker version, Alice aims at maximizing the number of edges she owns, while Breaker aims at minimizing it. In the Maker-Maker version, both players try to take more edges then their opponent.

We prove that surprisingly, computing the score in the Maker-Breaker version is Pspace-complete whereas in the Maker-Maker version, the relative score can be solved in polynomial time. In addition, for the Maker-Breaker version, we give some bounds on the score with an Erdös-Selfridge like proof, and we prove that the score on paths and cycles can be computed in polynomial time.

Reference

[1] P Erdös and J.L Selfridge, On a combinatorial game, Journal of Combinatorial Theory, Series A, 1973,

Edge-colored graphs without color-rich cycles

Tomáš Madaras, Alfréd Onderko

Pavol Jozef Šafárik University, Košice

Let $\mathcal{K}_i^\circ(G)$ be the maximum number of colors in an edge-coloring of a graph G such that each its cycle is colored with at most i colors. We give greedy-based degree-related general lower bounds on $\mathcal{K}_i^\circ(G)$ when $i \geq 3$ and discuss their sharpness. In particular, we provide exact values of $\mathcal{K}_3^\circ(G)$ and $\mathcal{K}_2^\circ(G)$ in the case when G is a 5-connected and a 3-connected graph, respectively. Furthermore, we show that the exact value of $\mathcal{K}_2^\circ(G)$ can be found by maximizing the number of components after deleting a 2-cut whenever $\kappa(G) = 2$, while $\mathcal{K}_2^\circ(G) = 2$ if G is highly connected. We also discuss the effectiveness of upper bounds on $\mathcal{K}_i^\circ(G)$ given by anti-Ramsey numbers for the cycles of length $i + 1$.

Transducing paths in graph classes with unbounded shrubdepth

Patrice Ossona de Mendez, Michał Pilipczuk, Sebastian Siebertz

CAMS CNRS/EHESS, Paris

Transductions are a general formalism for expressing transformations of graphs (and more generally, of relational structures) in logic. We prove that a graph class $\mathscr{C}$ can be FO-transduced from a class of bounded-height trees (that is, has *bounded shrubdepth*) if, and only if, from $\mathscr{C}$ one cannot FO-transduce the class of all paths. This establishes one of the three remaining open questions posed by Blumensath and Courcelle in [1] about the MSO-transduction quasi-order, even in the stronger form that concerns FO-transductions instead of MSO-transductions.

The backbone of our proof is a graph-theoretic statement that says the following: If a graph G excludes a path, the bipartite complement of a path, and a half-graph as semi-induced subgraphs, then the vertex set of G can be partitioned into a bounded number of parts so that every part induces a cograph of bounded height, and every pair of parts semi-induce a bi-cograph of bounded height. This statement may be of independent interest; for instance, it implies that the graphs in question form a class that is linearly χ-bounded.

Reference

[1] A. Blumensath and B. Courcelle, *On the monadic second-order transduction hierarchy*, Logical Methods in Computer Science **6** (2010), no. 2.

Invertible and pseudoinvertible simple connected graphs

Soňa Pavlíková, Daniel Ševčovič

Alexander Dubček University, Trenčín

In our talk we will analyse the Moore-Penrose pseudoinversion of block symmetric matrices. We will present sufficient conditions on the elements of a block matrix yielding an explicit block matrix form of its Moore-Penrose inversion. We will apply the explicit form for the spectral gap maximization with respect to off-diagonal elements of a given block matrix and we will use the results in the spectral graph theory where maximization of the spectral gap plays an important role in computational chemistry and analysis of stability of organic molecules.

The first author acknowledges support from the APVV Research Grants 17-0428 and 19-0308, and from the VEGA Research Grants 1/0206/20 and 1/0567/22.

Bisimplicial separators

Martin Milanič, Irena Penev, Nevena Pivač, Kristina Vušković

Computer Science Institute of Charles University, Prague

A *minimal separator* of a graph G is a set $S \subseteq V(G)$ such that there exist vertices $a, b \in V(G) \setminus S$ with the property that S separates a from b in G, but no proper subset of S does. For an integer $k \geq 0$, we say that a minimal separator is *k-simplicial* if it can be covered by k cliques, and we denote by $\mathcal{G}_k$ the class of all graphs in which each minimal separator is k-simplicial. A 2-simplicial separator is also called *bisimplicial*. Obviously, $\mathcal{G}_0 \subseteq \mathcal{G}_1 \subseteq \mathcal{G}_2 \subseteq \ldots$. Classes $\mathcal{G}_0$ and $\mathcal{G}_1$ are well understood: $\mathcal{G}_0$ is the class of all disjoint unions of complete graphs, and by a classical result of Dirac [1], $\mathcal{G}_1$ is precisely the class of all chordal graphs. In this talk, we present a number of structural and algorithmic results about classes $\mathcal{G}_k$ ($k \geq 2$), with a particular emphasis on $\mathcal{G}_2$.

We show that for each $k \geq 0$, the class $\mathcal{G}_k$ is closed under induced minors, and we also give a complete list of minimal forbidden induced minors for $\mathcal{G}_2$. We further show that, for $k \geq 1$, every nonnull graph in $\mathcal{G}_k$ has a k-simplicial vertex, i.e. a vertex whose neighborhood is the union of k cliques. We also present a decomposition theorem for diamond-free graphs in $\mathcal{G}_2$ (the *diamond* is the graph obtained from the complete graph on four vertices by deleting one edge, and a graph is *diamond-free* if none of its induced subgraphs is isomorphic to the diamond).

Relying on our structural results, as well as results from the literature, we obtain a number of algorithmic consequences, summarized in the table below (as usual, n is the number of vertices and m the number of edges of the input graph).

	diamond-free graphs in $\mathcal{G}_2$	$\mathcal{G}_2$	$\mathcal{G}_k$ ($k \geq 3$)
recognition	$\mathcal{O}(n(n+m))$	?	NP-hard
Maximum Weight Clique	$\mathcal{O}(n(n+m))$	$\mathcal{O}(n^4)$	NP-hard
Maximum Weight Stable Set	$\mathcal{O}(n^2(n+m))$	$\mathcal{O}(n^6)$	$\mathcal{O}(n^{2k+2})$
Vertex Coloring	$\mathcal{O}(n(n+m))$	NP-hard	NP-hard

Reference

[1] G.A. Dirac. On rigid circuit graphs. Abhandlungen aus dem Mathematischen Seminar der Universität Hamburg (1961), 25:71–76.

Twin-width and Limits of Tractability of FO Model Checking on Geometric Graphs

Petr Hliněný, Filip Pokrývka

Masaryk University, Brno

The complexity of the problem of deciding properties expressible in FO logic on graphs – the FO model checking problem (parameterized by the respective FO formula), is well-understood on so-called sparse graph classes, but much less understood on hereditary dense graph classes. Regarding the latter, a recent concept of twin-width [Bonnet et al., FOCS 2020] appears to be very useful. For instance, the question of these authors [CGTA 2019] about where is the exact limit of fixed-parameter tractability of FO model checking on permutation graphs has been answered by Bonnet et al. in 2020 quite easily, using the newly introduced twin-width. We prove that such exact characterization of hereditary subclasses with tractable FO model checking naturally extends from permutation to circle graphs (the intersection graphs of chords in a circle). Namely, we prove that under usual complexity assumptions, FO model checking of a hereditary class of circle graphs is in FPT if and only if the class excludes some permutation graph. We also prove a similar excluded-subgraphs characterization for hereditary classes of interval graphs with FO model checking in FPT, which concludes the line a research of interval classes with tractable FO model checking started in [Ganian et al., ICALP 2013]. The mathematical side of the presented characterizations – about when subclasses of the classes of circle and permutation graphs have bounded twin-width, moreover extends to so-called bounded perturbations of these classes.

Minimal induced subgraphs of two classes of 2-connected non-Hamiltonian graphs

Joseph Cheriyan, Sepehr Hajebi, Zishen Qu, Sophie Spirkl

University of Waterloo

Finding sufficient conditions for a class of graphs to be Hamiltonian is an old problem, with a wide variety of conditions such as Dirac's degree condition and Whitney's theorem on 4-connected planar triangulations. We discuss some past results on sufficient conditions for Hamiltonicity involving the exclusion of fixed induced subgraphs, and some properties of the graphs involved in such results. In 1981 Duffus, Gould, and Jacobson showed that any connected graph that does not contain a claw or a net as an induced subgraph has a Hamiltonian path. We aim to find an analogous result for Hamiltonian cycles. In particular, we would like to find a set of graphs S which are 2-connected, non-Hamiltonian, and every proper 2-connected induced subgraph is Hamiltonian such that every 2-connected S-free graph is Hamiltonian. In joint work with Joseph Cheriyan, Sepehr Hajebi, and Sophie Spirkl, we show that the classes of 2-connected split graphs and 2-connected triangle-free graphs can be characterised in this fashion.

On d-dimensional nowhere-zero r-flows on a graph

Davide Mattiolo, Giuseppe Mazzuoccolo, Jozef Rajník, Gloria Tabarelli

Comenius University, Bratislava

A d-dimensional nowhere-zero r-flow on a graph G, an (r, d)-NZF for short, is a flow where the value on each edge is an element of $\mathbb{R}^d$ whose Euclidean norm lies in the interval $[1, r - 1]$. Such a notion is a natural generalization of the well-known concept of circular nowhere-zero r-flow (i.e. $d = 1$). In this talk, we mainly consider the parameter $\phi_d(G)$, which is the minimum of the real numbers r such that G admits an (r, d)-NZF. For every bridgeless graph G. The 5-flow Conjecture claims that $\phi_1(G) \leq 5$, while a conjecture by Kamal Jain suggests that $\phi_d(G) = 1$, for all $d \geq 3$ [1].

Here, we address the problem of finding a possible upper-bound in the case $d = 2$. We show that, for all bridgeless graphs, $\phi_2(G) \leq 1 + \sqrt{5}$ and that the oriented 5-Cycle Double Cover Conjecture implies $\phi_2(G) \leq \Phi^2$, where Φ is the Golden Ratio. Moreover, we discuss some connections between this problem and some other well-known conjectures. Finally, we focus our attention on the cubic case: we propose a geometric method to describe an $(r, 2)$-NZF of a cubic graph in a compact way, and we apply it in some instances.

Reference

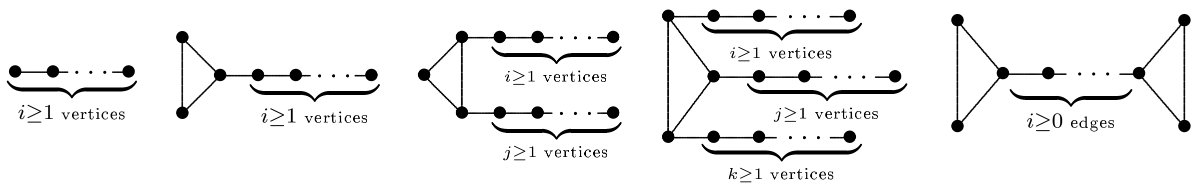
[1] http:garden.irmacs.sfu.ca?q=op/unit_vector_flows. Reference posted by M. DeVos on March 7th, 2007. Reference accessed on January 15, 2022.

Forbidden subgraphs implying Hamilton-connectedness

Zdeněk Ryjáček, Petr Vrána

University of West Bohemia, Pilsen

We consider the question for which pairs of connected graphs X, Y, every 3-connected (X, Y)-free graph G is Hamilton-connected (i.e., contains a hamiltonian path between any two vertices). The question has been studied since the early 90's, and known previous results imply that if X, Y are such graphs, then one of them, say X, is the claw $K_{1,3}$, and Y belongs to the following list: P_i for $i \le 9$, $N_{i,j,k}$ for $i + j + k \le 7$, $B_{i,j}$ for $i + j \le 7$, Z_i for $i \le 6$ or $i = 7$ and $|V(G)| \ge 21$, Γ_1, Γ_3, and Γ_5 for $|V(G)| \ge 21$ (for these graphs, see the figure below). There are many previously known partial results, however, among them, results for Γ_1, P_9 and $N_{1,2,3}$ are the only sharp ones; all the other known results deal with smaller values of i, j, k.



P_i $(i \ge 1)$ Z_i $(i \ge 1)$ $B_{i,j}$ $(i, j \ge 1)$ $N_{i,j,k}$ $(i, j, k \ge 1)$ Γ_i $(i \ge 0)$

We first develop techniques that allow to handle the sharp version of the problem, and then we settle the question in the affirmative for the graphs $N_{i,j,k}$ with $i + j + k = 7$, $B_{i,j}$ with $i + j = 7$, and Z_7 with one exceptional graph. All these results are sharp. Since the graphs Γ_1 and P_9 are already done, the only remaining candidates for Y are the graphs Γ_3 and Γ_5 for $|V(G)| \ge 21$.

The first part of the work (the results for the graphs $N_{i,j,k}$ with $i + j + k = 7$) is also a joint work with X. Liu, L. Xiong and X. Yang (Beijing).

Finding a Non-Shortest Path

Vibha Sahlot, Youngho Yoo

Charles University, Prague

Shortest Path is a well-studied problems with its many variants where the graph can be directed, undirected, with/ without negative edge weights, etc. We define this problem as follows. Given an unweighted graph[1] $G(V, E)$ and vertices $s, t \in V$, we find a minimum length path from s to t. This is in P using BFS.

We are interested in Non-Shortest Path (NSP). Given a graph $G(V, E)$ and $s, t \in V$, here we find a non-shortest path from s to t. It is inspired by the work of Bezáková et al. [1] where the authors try to find if any given undirected graph has an (s, t)-path of length at least $d_G(s, t) + k$ ($d_G(s, t)$ is the length of a shortest path from s to t). We name it Above Guarantee Path (AGP). They proved that AGP is in FPT. Hence NSP is in P. But the complexity of NSP is open for directed graphs (DNSP). Fomin et al. [2] studied directed version of AGP (DAGP) and proved that if 3−Disjoint Path (3-DP) is in P for a graph class (like DAGs, directed planar graphs, etc.), then DAGP is also FPT on that graph class. Hence, DNSP is in P on these graph classes. We refine it to a proposition that if 2−Disjoint Path (2-DP) is in P for a graph class then the NSP is also in P on that. We prove that given three pair of vertices and disjoint paths between every two pair, computing 3-DP is still NP-hard. Further we show if DNSP is in P then DAGP is in XP.

Working in a similar direction, we would like to find the class of graphs such that between any pair of vertices there are exactly k many paths that are of different lengths. For example, in undirected graphs, $k = 1$ for trees and $k = 2$ for odd cycles. This can also be asked for directed graphs, which can be further refined into all directed pairs or all pairs. We conjecture that no such graph exists for $k = 2$ in the first case. Similar questions can be asked for the case where between any pair of vertices there are at least k many paths of different lengths.

Reference

[1] Ivona Bezáková, Radu Curticapean, Holger Dell, Fedor V. Fomin, Finding detours is fixed-parameter tractable, SIAM J. Discret. Math., 33 (2019), pp. 2326–2345.

[2] Fedor V. Fomin, Petr A. Golovach, William Lochet, Danil Sagunov, Kirill Simonov, Saket Saurabh, Detours in Directed Graphs, STACS 2022, pp. 29,1-29,16.

[1] All the graphs considered are without parallel edges and self-loop.

The maximum number of copies of an even cycle in a planar graph

Zequn Lv, Ervin Győri, Zhen He, Nika Salia, Casey Tompkins, Xiutao Zhu

Alfréd Rényi Institute of Mathematics

We resolve a conjecture of Cox and Martin [1] by determining asymptotically for every $k \geq 2$ the maximum number of copies of C_{2k} in an n-vertex planar graph [2].

Reference

[1] C. Cox and R. R. Martin. Counting paths, cycles and blow-ups in planar graphs. *Journal of Graph Theory*, 10.1002/jgt.22838 (2022).

[2] Z. Lv, E. Győri, Zh. He, N. Salia and C. Tompkins, X. Zhu, The maximum number of copies of an even cycle in a planar graph, arXiv:2205.15810, 2022.

Geodesic transversal problem

Iztok Peterin, Gabriel Semanišin

Institute of Computer Science, Faculty of Science, P.J. Šafárik University, Košice, Slovakia

A set S of vertices of a graph G is a *geodesic transversal* of G if every maximal geodesic of G contains at least one vertex of S. The minimum cardinality of a geodesic transversal of G is denoted by gt and is called geodesic transversal number. These concepts were independently introduced in [1] and [2].

We describe fundamental properties of geodesic transversals and for two graphs G and H we deal with the behaviour of this invariant for the lexicographic product $G \circ H$ and join $G \oplus H$. We determine $\mathrm{gt}(G \oplus H)$ in terms of structural properties of the original graphs and describe $\mathrm{gt}(G \circ H)$ as a solution of an optimization problem concerning specific subsets of $V(G)$.

For details see [1] and [3].

Reference

[1] I. Peterin, G. Semanišin, On the Maximal Shortest Paths Cover Number. Mathematics. 2021; 9(14):1592. https://doi.org/10.3390/math9141592

[2] P. Manuel, B. Brešar, S, Klavžar, The geodesic-transversal problem. Appl Math Computat 2022: 413:126621. https://doi.org/10.1016/j.amc.2021.126621

[3] I. Peterin, G. Semanišin, Geodesic transversal problem for join and lexicographic product of graphs. Comp. Appl. Math. 41, 128 (2022). https://doi.org/10.1007/s40314-022-01834-1

Graph burning and non-uniform k-centers for small treewidth

Matej Lieskovský, Jiří Sgall

Computer Science Institute of Charles University, MFF UK, Praha

In an instance of the (uniform) k-center problem we are given a set of n nodes, a metric d defining distances between the nodes, and parameters k and r that tell us how many centers we are allowed to use and the radius of every center's reach, respectively. We must then decide if a set of k nodes can be selected as centers so that every node will be within distance r of some center. When minimizing r for a given k, 2-approximation algorithm is known and this is optimal unless P=NP.

A common generalization is the non-uniform k-center (NUkC) problem, where each of the k centers has its own radius. Current techniques work only with a limited number of different radii. The special case known as k-center with outliers uses only two radii r and 0; in this case there exists a 2-approximation algorithm (to minimize r) and this is optimal unless P=NP.

In the graph burning problem, each center gets a unique radius. Given an unoriented graph with unit-length edges and a single parameter g, our task is to cover the entire graph with g centers, each having a unique integer radius from 0 to $g - 1$. A 3-approximation algorithm for g has been given but no better result for general graphs is known. Graph burning was originally introduced to model the spread of a contagion through a network. In the original formulation, all nodes start unburned and in every time step fire spreads to all nodes that neighbour a burning node and then a single additional node is set on fire. The burning number g of a graph is defined as the number of time steps needed for all nodes to be on fire.

We approach the graph burning problem by restricting the graphs to be burned. For graph burning of linear forests (unions of disjoint paths) the problem is NP-hard and a PTAS is known.

We show that the non-uniform k-center problem is polynomial when parametrized by the number of different radii and treewidth. This extends the known exactly solvable cases of the non-uniform k-center problem; in particular this also solves the k-center with outliers on graphs of small treewidth exactly.

We then use this XP algorithm to design a PTAS for burning graphs of a constant treewidth, with a slight generalization that allows edge lengths. This result significantly improves previous results, as a PTAS was known only for linear forests, not even for trees.

Structured codes of graphs

Noga Alon, Anna Gujgiczer, János Körner, Aleksa Milojević, Gábor Simonyi

Alfréd Rényi Institute of Mathematics, Budapest
and
Department of Computer Science and Information Theory
Budapest University of Technology and Economics

We investigate codes the codewords of which are graphs that are identified with $0-1$ sequences on the edge set of a complete graph on n vertices in the natural way. The "structuredness"in the title refers to various requirements that are expressed by properties of the graphs we obtain as the symmetric difference of (the edge sets of) two graphs that appear in the code. We find the maximum possible size of such codes in various cases and show connections to such diverse areas of graph theory as perfect 1-factorizations and the theory of Turán numbers.

The talk is based on the paper [1].

Reference

[1] N. Alon, A. Gujgiczer, J. Körner, A. Milojević, G. Simonyi, Structured Codes of Graphs, submitted, arXiv:2202.06810 [math.CO] .

Perfect matchings in regular graphs

Yulai Ma, Davide Mattiolo Eckhard Steffen, Isaak H. Wolf

Paderborn University, Germany

Thomassen [Problem 1 in [1]] asked whether every r-edge-connected r-regular graph of even order has $r-2$ pairwise disjoint perfect matchings. We show that this is not the case if r is even.

It turns out that our methods are limited to the even case of Thomassen's problem. We then prove some equivalences of statements on pairwise disjoint perfect matchings in highly edge-connected regular graphs, where the perfect matchings contain or avoid fixed sets of edges.

Based on these results we relate statements on pairwise disjoint perfect matchings of 5-edge-connected 5-regular graphs to well-known conjectures for cubic graphs, such as the Fan-Raspaud Conjecture, the Berge-Fulkerson Conjecture and the 5-Cycle Double Cover Conjecture.

Reference

[1] C. Thomassen, Factorizing regular graphs, J. Combin. Theory Ser. B 141 (2020) 343–351

Random embedding of complete graph

Jesse Campion Loth, Kevin Halasz, Tomáš Masařík,
Bojan Mohar, Robert Šámal

Computer Science Institute, Faculty of Mathematics and Physics, Charles University, Praha

A random embedding of a graph is given by choosing randomly and independently a local rotation of edges incident with each of the vertices. We can then study properties of the resulting embedding, in particular the number of faces (equivalently, the genus of the embedding).

Random embeddings appear of sufficient interest not only in topological graph theory but also within several areas of pure mathematics and theoretical physics. The area was started by Stahl and Gross in the 80's. Stahl [1] proved that the expected number of faces of a random embedding of a complete graph K_n is at most $n + \log n$ and together with Mauk [2] they conjectured that the correct bound is $2 \log n + O(1)$. We improve his bound to $5 \log n + O(1)$ and also give a lower bound of the same asymptotic order.

Reference

[1] Saul Stahl. On the average genus of the random graph. *J. Graph Theory*, 20(1):1–18, August 1995. `https://doi.org/10.1002/jgt.3190200102`

[2] Clay Mauk and Saul Stahl. Cubic graphs whose average number of regions is small. *Discrete Mathematics*, 159(1-3):285–290, November 1996. `https://doi.org/10.1016/0012-365x(95)00089-f`

Structure of 3-stars in embedded graphs

Katarína Čekanová, Mária Maceková, Roman Soták, Zuzana Šárošiová

P. J. Šafárik University, Košice

For integers $k \geq 1$ and $1 \leq t \leq 3$, let $g(k,t)$ be the minimum integer such that every graph with girth at least $g(k,t)$, minimum degree at least 2 and no $(k+1)$-path consisting of vertices of degree 2, has a 3-vertex with at least t neighbors of degree 2. For the class of plane graphs there are many results concerning existence of a 3-vertex with specified number of 2-neighbors. Recently, Borodin and Ivanova established the value of $g(k,t)$ for all combinations of k and t (where $k \geq 1$ and $t \in \{1,2,3\}$). In the talk we present how the situation changes for the class of graphs embedded on a surface(s) with non-positive Euler characteristic.

Snarks with resistance n and flow resistance $2n$

Imran Allie, Edita Máčajová, Martin Škoviera

Comenius University, Bratislava

We examine the relationship between two measures of uncolourability of cubic graphs – their resistance and flow resistance. The resistance of a cubic graph G, denoted by $r(G)$, is the minimum number of edges whose removal results in a 3-edge-colourable graph. The flow resistance of G, denoted by $r_f(G)$, is the minimum number of zeroes in a 4-flow on G. Fiol et al. [1] made a conjecture that $r_f(G) \leq r(G)$ for every cubic graph G. We disprove this conjecture by presenting a family of cubic graphs G_n of order $34n$, where $n \geq 3$, with resistance n and flow resistance $2n$. For $n \geq 5$ these graphs are nontrivial snarks.

Reference

[1] M. A. Fiol, G. Mazzuoccolo, E. Steffen, Measures of edge-uncolourability of cubic graphs, Electron. J. Combin. 25 (2018), #P4.54.

Measure of simplicity of a tournament

Abderrahim Boussaïri, Imane Talbaoui, Soufiane Lakhlifi

Faculty of Science Aïn Chock, Casablanca, Morocco

The simplicity index of an n-tournament T is the minimum number $s(T)$ of arcs whose reversal yields a decomposable (non simple) tournament. Recall that an n-tournament T with vertex set V is decomposable (non simple) if there exist a subset M of V such that $2 \leq |M| \leq n-1$ and for every $x \in V \setminus M$, either $M \to x$ or $x \to M$. Müller and Pelant (1974) proved that $s(T) \leq \frac{n-1}{2}$, and that equality holds if and only if T is doubly regular. As doubly regular tournaments exist only if $n \equiv 3 \pmod 4$, $s(T) < \frac{n-1}{2}$ for $n \not\equiv 3 \pmod 4$ [1]. This property characterizes the class of doubly regular tournaments. In our work, we studied the class of n-tournaments with maximal simplicity index for $n \not\equiv 3 \pmod 4$.

Reference

[1] V. Müller, and J. Pelant , On strongly homogeneous tournaments, Czechoslovak Mathematical Journal. 24 (1974), 378–391.

[2] P. Erdős and J W. Moon, On sets of consistent arcs in a tournament, Canad. Math. Bull. 8 (1965), 269–271.

[3] J. Wallis, Some (1, -1) matrices, Journal of Combinatorial Theory, Series B. 10 (1971), 1–11.

[4] K B. Reid, and E. Brown, Doubly regular tournaments are equivalent to skew Hadamard matrices, Journal of Combinatorial Theory, Series A. 3 (1972), 332–338.

[5] C. Koukouvinos and S. Stylianou, On skew-Hadamard matrices, Discrete Mathematics 308 (2008), 2723–2731.

Erdős–Szekeres-type problems in the real projective plane

Martin Balko, Manfred Scheucher, Pavel Valtr

Charles University, Prague

We consider point sets in the real projective plane $\mathbb{RP}^2$ and explore variants of classical extremal problems about planar point sets in this setting, with a main focus on Erdős–Szekeres-type problems.

We provide asymptotically tight bounds for a variant of the Erdős–Szekeres theorem about point sets in convex position in $\mathbb{RP}^2$, which was initiated by Harborth and Möller in 1994. The notion of convex position in $\mathbb{RP}^2$ agrees with the definition of convex sets introduced by Steinitz in 1913.

For $k \geq 3$, an *(affine) k-hole* in a finite set $S \subseteq \mathbb{R}^2$ is a set of k points from S in convex position with no point of S in the interior of their convex hull. After introducing a new notion of k-holes for points sets from $\mathbb{RP}^2$, called *projective k-holes*, we find arbitrarily large finite sets of points from $\mathbb{RP}^2$ with no projective 8-holes, providing an analogue of a classical result by Horton from 1983. We also prove that they contain only quadratically many projective k-holes for $k \leq 7$. On the other hand, we show that the number of k-holes can be substantially larger in $\mathbb{RP}^2$ than in $\mathbb{R}^2$ by constructing, for every $k \in \{3, \ldots, 6\}$, sets of n points from $\mathbb{R}^2 \subset \mathbb{RP}^2$ with $\Omega(n^{3-3/5k})$ projective k-holes and only $O(n^2)$ affine k-holes. Last but not least, we prove several other results, for example about projective holes in random point sets in $\mathbb{RP}^2$ and about some algorithmic aspects.

The study of extremal problems about point sets in $\mathbb{RP}^2$ opens a new area of research, which we support by posing several open problems.

Coloring ordered graphs with excluded induced ordered matchings

Marcin Briański, James Davies, Bartosz Walczak

Jagiellonian University, Kraków, Poland

An ordered graph is a graph equipped with a total order on the vertices. We prove that for every ordered matching M, the class of ordered graphs excluding M as an induced ordered subgraph is χ-bounded. This generalizes the known fact that the class of outerstring graphs is χ-bounded and confirms a conjecture by István Tomon.

In general, coloring ordered graphs with excluded (induced) ordered subgraphs is an area that has been little explored so far. Our result is a step towards characterizing ordered graphs H such that the class of H-free ordered graphs is χ-bounded.

Circular Flows in Mono-directed Eulerian Signed Graphs

Jiaao Li, Reza Naserasr, Zhouningxin Wang, and Xuding Zhu

IRIF, Université Paris Cité, Paris

Given positive integers p, q where p is even and $p \geq 2q$, a circular $\frac{p}{q}$-flow in a mono-directed signed graph (G, σ) is a pair (D, f) where D is an orientation on G and $f : E(G) \to \mathbb{Z}$ satisfies that for each positive edge e, $q \leq |f(e)| \leq p - q$ and for each negative edge e, either $0 \leq |f(e)| \leq \frac{p}{2} - q$ or $\frac{p}{2} + q \leq |f(e)| \leq p - 1$, and the total in-flow equals the total out-flow at each vertex. This is the dual notion of circular $\frac{p}{q}$-coloring of signed graphs recently introduced in "Circular chromatic number of signed graphs. R. Naserasr, Z. Wang, and X. Zhu. Electronic Journal of Combinatorics, 28(2)(2021), #P2.44".

In this talk, we consider the signed bipartite analogs of Jaeger's circular flow conjecture and its dual, Jaeger-Zhang conjecture. We show that every $(6k-2)$-edge-connected Eulerian signed graph admits a circular $\frac{4k}{2k-1}$-flow and every signed bipartite planar graph of negative-girth at least $6k-2$ admits a circular $\frac{4k}{2k-1}$-coloring. We also provide recent results about circular flows in mono-directed signed graphs with high edge-connectivities and leave some further questions.

Computational Frameworks for Solving Graph Pebbling Problems

Oguzhan Colkesen, Dominic Flocco, Hammurabi Mendes,
Jonad Pulaj, Bryce Weidenbeck, Carl Yerger

Davidson College, Davidson, North Carolina, USA

We discuss recent results that extend computational models for computing pebbling numbers. In particular we will describe a new computational open source codebase for graph pebbling using a linear programming and weight function framework. We also give a new characterization of graph pebbling and rubbling as two player Stackelberg games via bilevel integer programming. Finally, we describe some preliminary results implementing a computational framework for each of these bilevel integer programming models.

Hamilton Cycles on Dense Regular Digraphs and Oriented Graphs

Allan Lo, Viresh Patel, Mehmet Akif Yıldız

University of Amsterdam, Netherlands

A (directed) cycle in a (directed) graph traversing all the vertices exactly once is called a Hamilton cycle. We prove that for every $\varepsilon > 0$ there exists $n_0 = n_0(\varepsilon)$ such that every regular oriented graph on $n > n_0$ vertices and degree at least $(1/4 + \varepsilon)n$ has a Hamilton cycle. This establishes an approximate version of a conjecture of Jackson from 1981. We also establish a result related to a conjecture of Kühn and Osthus about the Hamiltonicity of regular directed graphs with suitable degree and connectivity conditions.

Optimization of Scheduling of Incomplete Tournaments

Petr Kovář, Jakub Závada

Department of Applied Mathematics, Faculty of Electrical Engineering and Computer Science, VŠB-TU Ostrava, Ostrava

Methods of graph theory can be used for scheduling of sport tournaments. A tournament is represented by a graph G. Each participant of the tournament is represented by one vertex and each game by an edge between participants of the game. Vertex labeling of graphs is used to describe the strength of each participant. A tournament with n participants is called round robin if each participant plays every other in $n-1$ games. If each participant plays k, $k < n-1$, games, we call such tournament an incomplete tournament.

The focus of the talk is on the optimization of the scheduling of such incomplete tournaments, e.g. how to ensure comparable conditions for all participants when we need to schedule whole tournament ahead. The main part of the talk is focused on edge swapping. We use edge swapping to improve the properties of the scheduling without affecting the weight of any vertex. Another use of edge swapping is in creating tournaments with larger number of participant when we have a suitable scheduling for tournament with lower number of participants. Unfortunately, it is difficult to find a suitable subgraph in which we can swap edges without affecting the weight of any vertex. We show some subgraphs which make it possible, but also some general theorems describing the conditions under which such subgraph can/cannot exist.

Author Index